A TEXT-BOOK

ON

ROOFS AND BRIDGES.

PART I.

STRESSES IN SIMPLE TRUSSES.

BY

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PREFACE.

The course of instruction in roofs and bridges given to students of civil engineering in Lehigh University consists of four parts; first, the computation of stresses in roof trusses and in all the common styles of simple bridge trusses; second, the analysis of stresses by graphic methods; third, the design of a bridge, which includes the proportioning of details and the preparation of working drawings; and fourth, the discussion of cantilever, suspension, continuous and arched bridges. In the following pages the first part of this course is presented.

The plan adopted in arranging this text-book on the computation of stresses is similar to that followed in the author's "Mechanics of Materials." The principles and methods are first established, and then numerous numerical examples are fully worked out to illustrate them and their application to different forms of trusses, while a number of problems are stated as exercises for the student. As no work is so valuable to a student as that done by himself, each alternate leaf throughout the text has been left blank, so that his solutions of these problems may be recorded in permanent form.

In view of the importance of designing members for repeated stresses and shocks, particularly in bridge trusses, it has been thought well to compute in most cases the minimum as well as the maximum stresses. The range of stress thus becomes known for each member. It is believed that this feature will assist students in forming clear ideas as to the influence of the live load upon different members of the truss.

In the Chapter on roof trusses the fundamental principles are deduced, and directly applied to the computation of stresses caused by dead load, snow and wind. The simpler forms of roof trusses are well adapted to the elucidation of general principles, which in bridge trusses usually become of a special nature, owing to the parallelism of the chords.

In bridge trusses all methods of live loading are considered, beginning with that of a uniform load, passing to that of a locomotive excess over one or more panels, and concluding with that now in most general use—the actual locomotive wheel concentrations, followed by a uniform train load. Although the propriety of specifying typical locomotive wheel loads may perhaps be questioned, there can be no doubt but that, when once specified, the true static stresses should be computed for the given data, and not the approximate stresses from a so-called equivalent uniform load. The author has endeavored to present this subject in a simple manner and in accordance with the methods used in practice, and he acknowledges his indebtedness to the Phœnix Bridge Company for the convenient diagram for tabulating wheel moments.

MANSFIELD MERRIMAN.

LEHIGH UNIVERSITY, BETHLEHEM, PA.,
December, 1887

CONTENTS.

Chapter I.

STRESSES IN ROOF TRUSSES.

Chapter II.

HIGHWAY BRIDGE TRUSSES.

Chapter III.

RAILROAD BRIDGE TRUSSES.

A TEXT-BOOK

ON

ROOFS AND BRIDGES.

CHAPTER I.

STRESSES IN ROOF TRUSSES.

ART. 1. DEFINITIONS.

A truss is a structure arranged to carry loads in such a manner that each principal member is subject only to stress in the direction of its length, that is, to a tensile or to a compressive stress. The points where these members meet are called 'joints,' and the rivets, pins, or other connections which form the joints are subject to shearing, or to combined stresses of shear, compression and flexure. The simplest form of a roof truss is a triangle, such as shown in Fig. 1, consisting of two inclined compression members and a horizontal tension member, the load P being applied at the peak.

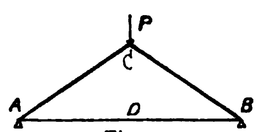

Fig. 1.

In order that a truss may perfectly conform to the above definition it is necessary that the loads should be supported only at the joints, for, if placed at intermediate points on the members, flexural stresses will be produced. Trusses are sometimes built with loads so placed, but it is not regarded as the best design, and the flexural effects must be carefully computed.

It is further necessary that all the elementary figures in a truss should be triangles, since a triangle cannot change its shape without altering the lengths of its sides, and hence the deformation of the truss under loads will be due only to the slight alterations in the lengths of the members caused by the stresses. A rectangular or polygonal figure, on the other hand, if loaded at one or more joints, can change its shape without altering the lengths of its sides, and hence would be unstable.

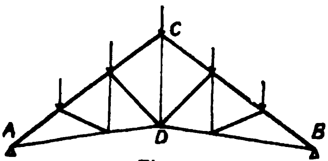

Fig. 2.

Members which take compression are called 'struts,' 'columns,' or 'posts.' Members which take tension are called 'ties.' The upper member, or line of members *A C* (Figs. 1 and 2), is designated as the 'upper chord' or sometimes as the 'main rafter,' and the lower part *ADB* as the 'lower chord,' or 'tie rod.' The members connecting the upper with the lower chord are termed 'braces;' some of these are in tension and others in compression.

The fundamental principles of statics, as set forth in books on 'Theoretical Mechanics' and as exemplified in 'Mechanics of Materials,' serve to determine the stresses in trusses due to given loads and to investigate the strength of members and joints. The following problems are now to be solved by the student, referring, if necessary, to books on these subjects.

Prob. 1. In Fig. 1, the span *AB* is 24 feet and the rise *CD* is 12 feet. Find the stresses in the three members due to a load *P* of 8 000 pounds.

Prob. 2. The span in Fig. 1 is 24 feet and the rise is 6 feet. Find the stresses due to a load of 8 000 pounds at the peak.

Prob. 3. If *AB*, *AC* and *BC* are timbers, each 3 × 4 inches, find the stresses per square inch for the case of the last problem. (See Mechanics of Materials, Arts. 5 and 59). Are the unit-stresses too high or too low?

ART. 2. LOADS ON ROOF TRUSSES.

The loads to be considered in discussing a truss are of four kinds: the weight of the truss itself, the weight of roof-covering, the snow, and the wind.

The weight of the truss depends upon the span, the distance apart of the adjacent trusses in the roof, the weight of the roof covering, and other elements of design. This weight can only be ascertained by the records of experience, and in RICKER'S 'Construction of Trussed Roofs,' page 46, is a table deduced from data given by different authorities which seems to afford the best figures now attainable. The following formulas give results approximately agreeing with those found by the use of this table. Let l be the span in feet, a the distance in feet between adjacent trusses, and W the approximate weight of one truss in pounds. Then,

$$\text{For wooden trusses,} \quad W = \tfrac{1}{2}al\left(1 + \tfrac{1}{10}l\right), \qquad (1)$$
$$\text{For wrought iron trusses,} \quad W = \tfrac{3}{4}al\left(1 + \tfrac{1}{10}l\right).$$

The wooden trusses are to have wrought iron tension members, in accordance with the usual practice, and it is seen that they are materially lighter than the wrought iron trusses. For example, if $l = 100$ feet and $a = 12$ feet, the formulas give about 6 600 pounds for a wooden and about 9 900 pounds for a wrought iron truss.

The roof covering consists of the exterior 'shingling' of tin, slate, tiles, corrugated iron, or wooden shingles, resting usually upon timber 'sheathing,' which is supported by 'purlins,' or beams, running longitudinally between the trusses and fastened to them at the upper joints. In large roofs the sheathing is laid upon 'rafters' parallel to the upper chord, the rafters resting upon the purlins. The actual weight of the roof covering, rafters, and purlins is to be determined only by computation for each particular case, but the following values will serve for preliminary designs and approximate computations. The weights given are in all cases per square foot of roof surface.

For shingling—tin, 1 pound; wooden shingles, 2 or 3 pounds; iron, 1 to 3 pounds; slates, 10 pounds; tiles, 12 to 25 pounds.
For sheathing—boards 1 inch thick, 3 to 5 pounds.
For rafters—1.5 to 3 pounds.
For purlins—wood, 1 to 3 pounds; iron, 2 to 4 pounds.
Total roof covering—from 5 to 35 pounds, per square foot of roof surface.

The snow load varies with the latitude, being about 30 pounds per horizontal square foot in northern New England, Canada, and Minnesota, about 20 pounds in the latitude of New York City and Chicago, about 10 pounds in the latitude of Baltimore and Cincinnati, and rapidly diminishes southward. On roofs having an inclination to the horizontal of 60 degrees or more this load may be neglected, as it might be expected that the snow would slide off.

The wind load is variable in direction and intensity, and often injurious in its effects. As it is very customary, however, to design small roofs without considering the wind, the subject will be deferred until Art. 13.

For the purpose of securing uniformity in the solution of the examples and problems given in this book, the following average values will be used, unless otherwise specified:

For the truss weight—compute from formulas (1).
For the roof covering—12 pounds per square foot of roof surface.
For the snow load—15 pounds per square foot of horizontal area.

Prob. 4. A wrought iron roof truss, like Fig. 2, has its span 80 feet and its rise 30 feet. The distance between trusses is 13 feet, 6 inches, center to center. Find the approximate weight of

the truss, the weight of the roof covering, and the snow load upon it.

Prob. 5. A wooden roof truss, like Fig. 3, has the span 60 feet, rise at peak 30 feet, rise at hip 20 feet, horizontal distance of hip from peak 20 feet, distance between trusses 12 feet. Find the approximate weights of the truss, roof covering and snow load.

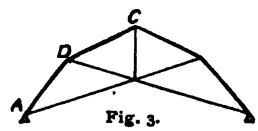

Fig. 3.

ART. 3. APEX LOADS AND REACTIONS.

The weight of the snow and of the roof covering is brought, as has been shown, by the purlins to the joints or 'apexes' of the upper chords of the roof truss. The weight of the truss itself is also generally regarded as concentrated at the same points, the larger part of it being in fact actually there applied. At each apex of the rafter there is therefore a load, called an 'apex load' and these loads produce stresses in the truss. The loads together with the reactions of the supports constitute, in fact, a system of forces held in equilibrium by the stresses in the members of the truss.

Having found the weight of the truss, roof covering and snow load, the apex loads are easily determined by dividing the total load by the number of divisions in the upper chords, if these be of equal length. These divisions AD, DC, etc., are called 'panels,' or sometimes 'bays.' Thus in Fig. 4, the apex loads at C and D

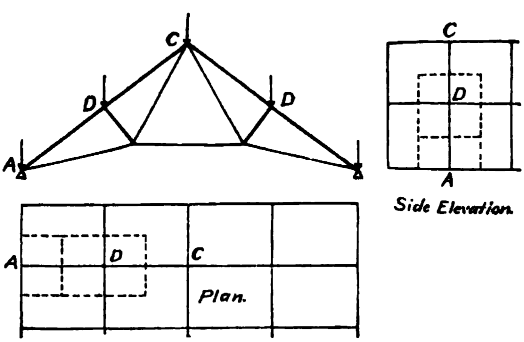

Fig. 4.

are each one-fourth of the total load. At the supports the apex loads are but one-half those at *C* and *D*. The apex load is often called the 'panel load.'

If the panels be of unequal length the load at any apex is found by considering that the weights there brought by the purlins are those upon a rectangle extending in each direction halfway to the adjacent apexes, as illustrated in Fig. 4.

The reactions of the supports are equal and each one-half of the total load, provided the two halves of the truss are symmetrical. For unsymmetrical roof-trusses the reactions are found in the same manner as for concentrated loads on a beam.

For example, take the case in Prob. 5. Here *AD* and *DC* are found to be 22.36 feet. The truss weight from formula (1) is 2 520 pounds. The weight of the roof covering on *AD* or *DC* is $12 \times 12 \times 22.36 = 3\,220$ pounds. On *AD* there is no snow, as its inclination is greater than 60 degrees; on *DC* the snow load is $15 \times 12 \times 20 = 3\,600$ pounds. The apex loads now are,

	Truss.	Covering.	Snow.	Total.
At *C*,	630	3 220	3 600	7 450
At *D*,	630	3 220	1 800	5 650
At *A*,	315	1 610	0	1 925

The total weight of truss, covering, and snow hence is 22 600 pounds, and each reaction is 11 300 pounds.

Prob. 6. Find the apex loads and the reactions for the case of Prob. 4.

Prob. 7. A wrought iron truss, like Fig. 3, has the span 50 feet, rise at peak 25 feet, rise at hip 18 feet, horizontal distance from hip to peak 17 feet, distance between trusses 12 feet. Find the apex loads and the reactions.

Art. 4. Relations between External Forces and Internal Stresses.

The applied loads and the reactions of the supports are held in equilibrium by the internal stresses in the members of the

truss. If any section *mn* be imagined to be drawn cutting the truss, and forces S_1, S_2, etc., be applied to the members cut which are equal in intensity and direction to the stresses in those members, then the equilibrium will be undisturbed. The applied forces on one side of the section R, P_0, P_1, etc., together with the internal stresses S_1, S_2, etc., are hence a system in static equilibrium. Therefore, the important principle,

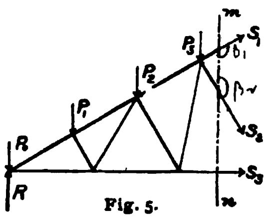

Fig. 5.

> The internal stresses in any section hold in equilibrium the external forces on either side of the section.

The fundamental conditions of static equilibrium are three in number, and hence are sufficient to determine the unknown stresses if these be not greater than three.

The fundamental conditions of static equilibrium of a system of forces in a plane are the following: (See text-books on Elementary Mechanics.)

$$\Sigma \text{ horizontal components} = 0,$$
$$\Sigma \text{ vertical components} = 0,$$
$$\Sigma \text{ moments} = 0.$$

These conditions state the relations between the internal stresses in any section and the external forces on either side of that section.

From these conditions three equations may be written for any particular case. For example, in Fig. 5 let S_3 be horizontal, and β_1 and β_2 be the angles made by S_1 and S_2 with the vertical. Then from the first condition,

$$S_1 \sin \beta_1 + S_2 \sin \beta_2 + S_3 = 0,$$

and from the second,

$$R - P_0 - P_1 - P_2 - P_3 + S_1 \cos \beta_1 - S_2 \cos \beta_2 = 0.$$

For the third condition a center of moments is to be selected; this may be taken at any point. If it be taken at the apex P_3, the moments of P_3, S_1 and S_2 are zero, and the equation is

$$(R - P_0)\, r - P_1 p_1 - P_2 p_2 - S_3 s_3 = 0,$$

where r, p_1, p_2 and s_3 denote the lever arms of R, P_1, P_2 and S_3.

It is best to regard the unknown stresses as tensile and to represent them by arrows pointing away from the section, as in Fig. 5. Then state the equations and find the numerical values of the stresses; if these values are positive the supposition as to direction is correct and the forces are tensile, but if negative the direction should be reversed, or the forces are compressive.

Prob. 8. In Fig 1, the load P is 12 000 pounds, the span AB is 30 feet and the rise CD is 16 feet. Find the stresses in AC and AD, using the first and second conditions only. Find the stress in AD, using the third condition only.

Prob. 9. In the wooden truss of Fig. 4, the span is 40 feet, rise of peak 15 feet, rise of tie rod 3 feet, distance between trusses 12 feet, and mean loads as specified in Art. 2. Find the apex loads, the reactions, and the stress in the horizontal part of the main tie rod.

ART. 5. THE METHOD OF MOMENTS.

The principle of moments is merely the third condition of static equilibrium, that the algebraic sum of the moments of all forces (Fig. 5) is zero, wherever the center of moments be taken. By the successive application of this principle, using different centers of moments, the three unknown stresses may be found without the necessity of using the first and second conditions of equilibrium. Thus, for Fig. 5, an equation of moments containing only S_3 was written in the last Article. An equation containing only S_2 may in like manner be written by taking the center of moments at the point of support, for then the moments

of S_1 and S_3 are zero. And in general the stress in any member may be found by the method of moments as follows:

> Draw a section cutting three members. To find the stress in one of these members, take the center of moments at the intersection of the other two members, state the equation of moments between the stress and the applied forces on the left of the section, and solve it for the unknown stress.

If the section drawn should cut but two members, the center of moments for finding the stress in one of them may be taken at any point upon the other.

For example, take the truss in Fig. 6, where the span is 36 feet, rise 14 feet, and apex loads as shown, the member S_5 being normal to the main rafter or upper chord at its middle point. To find the stress S_2, draw a plane cutting S_2, S_6 and S_4, let the direction of S_2 be away from the section, and take the center of moments at the peak. Then

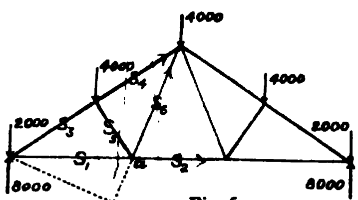

Fig. 6.

$$(8\,000 - 2\,000)\,18 - 4\,000 \times 9 - S_2 \times 14 = 0,$$

from which $S_2 = +\ 5\,140$ pounds, that is, tension. Similarly to find S_1, take the center of moments at the apex above it, then the lever arm of S_1 is 7 feet, and

$$(8\,000 - 2\,000)\,9 - S_1 \times 7 = 0, \text{ whence } S_1 = +\ 7\,710 \text{ pounds.}$$

For S_4 draw a section cutting S_4, S_5 and S_1, take the center of moments at a, and find the lever arms for this center, then

$$(8\,000 - 2\,000)\,14.44 - 4\,000 \times 5.44 + S_4 \times 8.87 = 0,$$

from which $S_4 = -\ 7\,200$ pounds, that is, compression. For S_3 the center is also at a and S_3 is found to be $-\ 9\,770$ pounds.

For S_5 the section cuts S_1, S_5 and S_4, the center is at the support, and

$$4\,000 \times 9 + S_5 \times 11.4 = 0, \text{ whence } S_5 = -\,3\,160 \text{ pounds.}$$

For S_6 the section cuts S_2, S_6 and S_4, the center is at the support, the lever arm is 14.0 feet, and

$$4\,000 \times 9 - S_6 \times 14.0 = 0, \text{ whence } S_6 = +\,2\,570.$$

The stresses in the right hand part of the truss are evidently the same as those just found for the left hand part.

Prob. 10. A wooden truss, like Fig 6, has a span of 40 feet, a rise of 15 feet, and the distance apart of trusses 12 feet. Find total apex loads, reactions, and the stresses in all the members.

ART. 6. LEVER ARMS.

The method of moments serves to determine the stresses in all the members of any truss, provided a section can be drawn cutting less than four members. The only difficulty in the application of this lies in the determination of the lever arms of the stresses and the applied forces. These can always be found from the given data by the use of geometry and trigonometry, but the computation is sometimes laborious. The principle of similar triangles will in general be found useful and fruitful for this purpose. General rules need not, and indeed cannot, be given, applicable to all forms of trusses, but it will be found advisable to check the values obtained by making a drawing of the truss and measuring the lever arms by scale. The lever arms may, in fact, be found by this method with sufficient precision without the necessity of computation, if the drawing be carefully made to a proper scale.

On account of the difficulty of computing the lever arms it is often customary to find only a part of the stresses by the method of moments, and to use the first and second conditions of equi-

librium for the determination of the others. This will be exemplified in the next Article.

Prob. 11. A roof truss, like Fig. 4, has its span 40 feet, rise of peak 15 feet, rise of tie rod 4 feet, the brace at D being normal to the main rafter at its middle point. State the equation of moments for each of the unknown stresses, and find the numerical values of all the lever arms.

Prob. 12. In the truss of Fig. 3 there is one member whose stress cannot be found by moments according to the method as above explained. Why? Explain how it can be found by moments.

ART. 7. THE METHOD OF RESOLUTION OF FORCES.

The principle of this method is embraced in the first and second conditions of static equilibrium as stated in Art. 4. Thus, in Fig. 7, if β_1, β_2, β_3 and β_4 be the angles made by S_1, S_2, S_3 and S_4 with the vertical, we have for a section cutting S_1, S_4 and S_3, the two equations

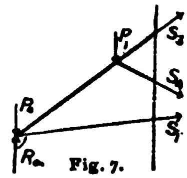

Fig. 7.

$$S_1 \sin \beta_1 + S_4 \sin \beta_4 + S_3 \sin \beta_3 = 0.$$

$$R - P_0 - P_1 + S_1 \cos \beta_1 + S_3 \cos \beta_3 - S_4 \cos \beta_4 = 0.$$

Also if a section be drawn cutting S_2, S_3 and S_4, as in Fig 8, we have

$$S_3 \sin \beta_3 + S_4 \sin \beta_4 - S_2 \sin \beta_2 = 0,$$

$$S_3 \cos \beta_3 - P_1 - S_4 \cos \beta_4 - S_2 \cos \beta_2 = 0.$$

Now, if in either of these cases one of the unknown stresses be first found by the method of moments, we have two equations containing two unknown quantities whose solution will determine the other two stresses. The angle made

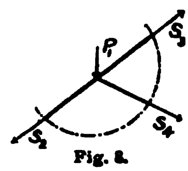

Fig. 8.

by the member with the vertical is always to be taken as acute, so that both its sine and cosine are positive.

By the successive application of this method, beginning with the two members which meet at the support, it is possible to find all the stresses without using the principle of moments.

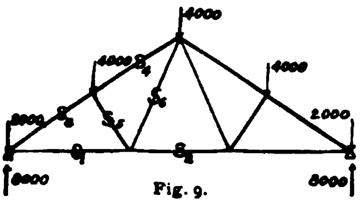

Fig. 9.

For example, take the truss in Fig. 9, which is the same as discussed in Art. 5, where the span is 36 feet and the rise 14 feet. From the given data, we find,

$$\sin\beta_1 = \sin\beta_2 = 1, \qquad \cos\beta_1 = \cos\beta_2 = 0,$$
$$\sin\beta_3 = \sin\beta_4 = 0.790, \qquad \cos\beta_3 = \cos\beta_4 = 0.614,$$
$$\sin\beta_5 = 0.614, \qquad \cos\beta_5 = 0.790,$$
$$\sin\beta_6 = 0.247, \qquad \cos\beta_6 = 0.969.$$

Now cutting S_1 and S_3, we have the two equations,

$$S_1 + S_3 \times 0.79 = 0 \quad \text{and} \quad 6\,000 + S_3 \times 0.614 = 0,$$

from which $S_3 = -9\,770$ and $S_1 = +7\,720$ pounds. Next cutting S_1, S_4 and S_5, we have

$$S_1 + S_5 \times 0.614 + S_4 \times 0.790 = 0,$$
$$6\,000 - 4\,000 - S_5 \times 0.790 + S_4 \times 0.614 = 0.$$

Inserting the value of S_1 in this and solving, we find that $S_4 = -7\,200$ and $S_5 = -3\,160$, as before. To find S_2 and S_6 a section may cut S_2, S_6 and S_4, or one may be drawn cutting S_2, S_6, S_5 and S_1.

Prob. 13. Find the stresses in the members of Fig. 1 by this method, taking the load P as 10 000 pounds, the span 24 feet and the rise 12 feet.

Prob. 14. Find the stresses in S_4 (Fig. 9) by using a section cutting the three members S_3, S_4 and S_5.

ART. 8. REMARKS.

The words 'horizontal' and 'vertical' used in stating the first and second conditions of equilibrium have thus far been used in their literal sense, but really, as shown in Analytical Mechanics, any two rectangular directions may be used instead, the general principle being that 'the sum of all the components must be zero for any given direction' in order to insure equilibrium. The 'horizontal' may hence be taken as any direction, the 'vertical' being at right angles to it. By choosing properly the direction in which to resolve the forces the determination of stresses may often be simplified. Thus, in Fig. 9, to find S_5, draw a section cutting S_3, S_4 and S_5, and resolve the forces into a direction parallel with S_5. Taking α as the acute angle between the load and S_4 the equation is $4\,000 \sin \alpha + S_5 = 0$, whence $S_5 = -3\,160$.

The same remarks made in Art. 6 regarding lever arms apply also to the determination of the sines and cosines necessary for the method of resolution of forces. In finding these it will rarely be advantageous to use tables, but it will be best to compute them directly by the geometric relations of the figure and to check the results, if thought necessary, by a skeleton diagram of the truss drawn to scale. Three figures in the sine and cosine are sufficient, or for very important cases four, so that the stresses may be accurately computed to the nearest hundred pounds. For example, to find $\sin \beta_3$ we have

$$\sin \beta_3 = \frac{9}{\sqrt{9^2 + 7^2}} = \frac{9}{11.402} = 0.7895 + = 0.790 \text{ to three figures.}$$

The student will have observed that the reaction and the half-apex load acting at the support are equivalent to an upward force equal to their difference. This upward force is the reaction due to the other apex loads, so that the apex loads at the supports may be entirely omitted from consideration if desired. Thus, in the two cases shown in Fig. 10, the same effect is produced on the beam or truss by the second system of loads as by the first, for the loads at the end are directly borne by the supports.

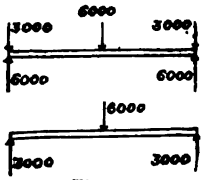

Fig. 10

Usually in computing the stresses it will be best to use the method of moments for all pieces whose lever arms can be easily computed, and then to employ the method of resolution of forces for the others, selecting the direction of resolution so as to produce the simplest equations.

Prob. 15. A truss, like Fig. 4, has 40 feet span, 15 feet rise at peak, and middle tie rod raised 4 feet. The apex loads at D and C are each 8 000 pounds. Find the stresses in all the members.

Prob. 16. In a truss, like Fig. 1, $AD = a$, $DB = b$ and $CD = h$. Find the stresses in the members caused by a load P at the peak C.

ART. 9. DEAD LOAD STRESSES.

The weight of a roof truss and of the roof covering borne by it is called the 'dead load,' or 'permanent load.' To find the stresses caused by the dead load it is only necessary to proceed as above, omitting the snow load from consideration. For example, the truss shown in Fig. 11 may be discussed. The span is 100 feet, rise 20 feet, each rafter of upper chord divided into four panels, from which verticals are dropped upon the lower chord as shown.

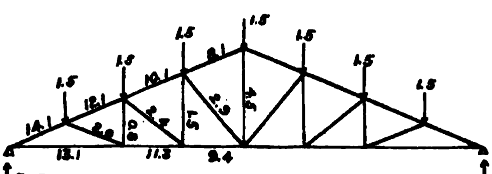
Fig. 11.

The truss being built with wooden compression members and iron tension members, its weight by formula (1) is found to be 7 150 pounds. The distance apart of trusses is 13 feet, and the

load per square foot of roof surface, 12 pounds; hence each apex load is

$$\frac{7\,150}{8} + 12 \times 13 \times \sqrt{12.5^2 + 5^2} = 2\,994 \text{ pounds} = 1.5 \text{ short tons},$$

and each effective reaction is 5.25 short tons.

The stresses in the lower chords are best found by moments, the lever arms being 5, 10 and 15 feet to the centers in the upper chord. The verticals (except the center one) are also best found by moments, taking the center at the support. For the upper chords the method of resolution of forces may be used, as also for the diagonals. The upper chord and diagonals are found to be in compression, while the lower chord and the verticals are in tension. This form of triangular roof truss is well adapted to the combined use of wood and iron, and is extensively built.

The following are the equations for finding the stresses in a few of the members. For the lower chord panel nearest the center,

$$5.25 \times 37.5 - 1.5\,(25 + 12.5) - S \times 15 = 0,$$

For the second vertical,

$$-\,1.5\,(25 + 12.5) + S \times 37.5 = 0,$$

For the second diagonal brace,

$$11.3 - 9.4 + S \times 0.781 = 0.$$

The dead load stresses, as thus found, are written on the diagram in short tons, the tensile members being drawn light and the compressive ones heavy.

Prob. 17. An iron truss, like Fig. 11, has 100 feet span, 18 feet rise, and distance between trusses 14 feet. Find all the stresses in short tons.

ART. 10. SNOW LOAD STRESSES.

While the dead load is estimated per square foot of inclined roof surface, the snow load is taken per square foot of horizontal area, since no more snow can fall upon an inclined surface upon its horizontal projection. Now, if the upper chords or main truss rafters are straight from support to peak, as in Fig. 11, the area of roof surface for any panel bears a constant ratio to its horizontal projection. Consequently the snow apex loads are all equal, and therefore, the stresses due to the snow are to the dead load stresses in the same ratio as the corresponding apex loads. Thus, for Fig. 11, the dead apex load was 1.5 tons, and the snow apex load is

$$15 \times 13 \times 12.5 = 2\,438 \text{ pounds} = 1.219 \text{ tons.}$$

Therefore, the snow load stresses may be found by multiplying the dead load stresses by $\frac{1219}{1500}$, that is, by 0.813.

If the upper chords are not straight from supports to peak the snow apex loads will not bear a constant ratio to the dead apex loads, and the snow load stresses must be independently determined. For instance, in the crescent truss whose dimensions are shown in Fig. 12, the snow apex loads are, for trusses 12 feet apart, $15 \times 16 \times 12 = 2\,880$ pounds at the peak, and $15 \times 12 \times 12 = 2\,160$ pounds at the hip, or 1.44 and 1.08 short tons. The stresses due to these loads may now be found either by the method of moments or by the method of resolution of forces.

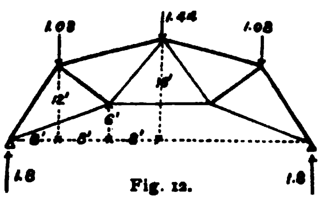

Fig. 12.

Prob. 18. Find the stress caused by the snow in the horizontal tie of Fig. 12. *Ans.* 2.59 tons tension.

Prob. 19. Find the stress caused by the snow in the middle brace of Fig. 12. *Ans.* Lever arm = 25 ft. and stress = 0.12 tons compression.

ART. 11. AMBIGUOUS CASES.

When the members of a truss are so arranged that it is impossible at certain places to pass a section cutting less than four pieces a difficulty or ambiguity may arise, since the three conditions of

equilibrium can determine but three unknown quantities. In such cases a fourth condition is sometimes found in the symmetry of the truss and loads. The common form known as the Fink roof truss furnishes an example of apparent ambiguity. Here the rafter *ae* is divided into four equal parts and normal to it are drawn the struts *bf*, *cg*, and *dh*, while all the other members are ties. Now, for the member *ch*, no section can be drawn cutting less than four pieces. But as *bf* and *dh* are symmetrically situated with reference to the loads their stresses are equal, and the same is true for *cf* and *ch*; hence, as the stress in *cf* can be found, that in *ch* is known.

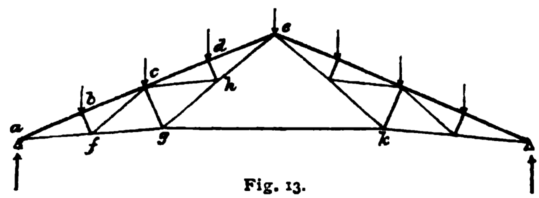

Fig. 13.

The stress in *ch* can also be determined in another way. First find the stress in *gk* by moments, then draw a section cutting *cd*, *ch*, *gh* and *gk*, and state an equation taking the center of moments at the peak. This equation contains the stresses in *ch* and *gk*, but the latter is known, and hence the former is easily obtained.

Prob. 20. In Fig. 13 take the span 80 feet, rise of peak 17 feet, rise of tie 2 feet, apex loads each 2.525 tons. Find the stresses in all members. *Ans.* $gk = +13.4$, $gh = +9.6$, $ch = +3.8$, $cd = -26.6$, $dh = -2.4$, etc.

ART. 12. THE ENDS OF ROOF TRUSSES.

Roof trusses of short span, and particularly wooden trusses, have generally both ends firmly 'fixed' to the supporting walls. But iron trusses and usually all trusses of large span have only one end fastened, while the other is 'free' or merely supported, so that it may move horizontally in the direction of the plane of the truss. This construction is adopted in order that the truss may expand and contract under changes of temperature and thus the stresses due to this cause be avoided. (Mechanics of Materials, Art. 73.)

There are three methods of arranging the supported end of the truss, 1st, it may rest upon a smooth iron plate upon which it slides; 2d, it may be arranged with a rocker as at *A*, Fig. 14; or 3d, it may rest upon rollers as at *B*. The last method is the one most generally employed. The first method in the case of heavy roofs is objectionable on account of the large frictional resistance to sliding.

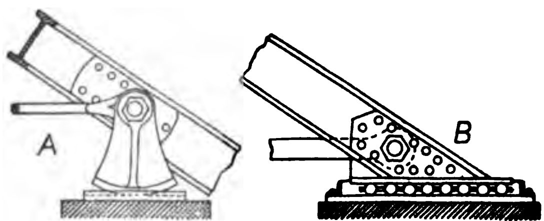

Fig. 14.

Prob. 21. In the roof of Fig. 1 the ends are fastened to the walls. If the stress in the wrought iron tie rod is 8 000 pounds per square inch at 70° Fahr., what will be the stress at 40° Fahr.?

ART. 13. WIND LOADS.

The pressure produced by the wind depends upon its velocity, being about 1 pound per square foot for a velocity of 15 miles per hour, about 5 pounds for 30 miles, about 18 pounds for 60 miles, and probably 50 pounds for a very violent hurricane at 100 miles per hour. For roof and bridge computations the pressure is usually taken at 40 pounds per square foot of vertical surface, the wind being supposed to move horizontally.

In England and to a slight extent in this country the effect of the wind is computed by placing a vertical load of 20 to 40 pounds per square foot on one-half the roof only and finding the

stresses due to this load. This method is defective, because the action of the wind is usually horizontal rather than vertical.

The action of the wind on an inclined roof surface is not fully understood, but experiments indicate that the resultant effect of a horizontal wind on an inclined surface may be represented by a normal force varying with the roof inclination. The following values deduced from HUTTON'S experiments give the normal pressure per square foot for a horizontal wind pressure of 40 pounds per square foot for different inclinations of the roof surface:

INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.
5°	5.1	25°	22.6	45°	36.0
10°	9.6	30°	26.5	50°	38.1
15°	14.2	35°	30.1	55°	39.4
20°	18.4	40°	33.3	60°	40.0

For all inclinations greater than 60° the normal pressure per square foot is 40 pounds. If the horizontal wind pressure should be assumed lower or higher than 40 pounds the normal pressures may be decreased or increased in the same ratio. For intermediate inclinations interpolations may be made in the table.

The wind apex loads are next to be found. For example, let Fig. 15 represent a truss of the dimensions shown, the distance between trusses being 12 feet. The inclination of *ab* is found to be 56° 19′, and that of *bc* 14° 02′, and hence from the above table the normal wind pressures per square foot are 39.5 and 13.3 pounds respectively. The total normal wind pressure on *ab* is then

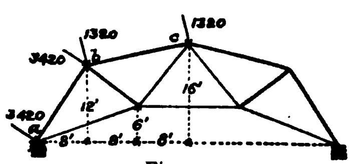

Fig. 15.

$$39.5 \times 12 \times \sqrt{64 + 144} = 6\ 840 \text{ pounds},$$

one-half of which is applied at *a* and one-half at *b*, as shown.

In the same way, the wind upon *bc* brings at *b* and *c* two normal apex loads, each of 1 320 pounds.

Prob. 22. Find the wind apex loads for the Fink truss of Prob. 20 and Fig. 13, the trusses being 16 feet apart between centers.

ART. 14. REACTIONS DUE TO WIND LOADS.

The reactions caused by the wind are inclined, the horizontal components of which tend to push over the walls of the building. It will be necessary to distinguish two cases, the first when both ends of the truss are fixed, and the second when one end is free to move.

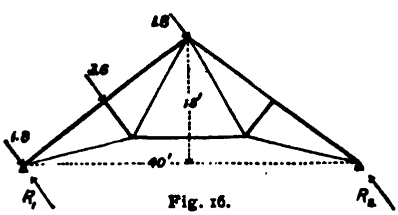

Fig. 16.

Let Fig. 16 represent a truss with both ends fixed, its span being 40 feet and its rise 15 feet, and the wind apex loads 1.8, 3.6 and 1.8 tons, as shown. The reactions R_1 and R_2 are parallel to the wind loads, since the sum of the components of all exterior forces in that direction must vanish. If θ be the angle made by the upper chord with the horizontal, and the center of moments be at the left support, we have

$$R_2 \times 40 \cos \theta - 1.8 \times 20 \sec \theta - 3.6 \times 10 \sec \theta = 0,$$

from which, since $\cos \theta = \frac{4}{5}$, the value of R_2 is 2.8125. In the same way by taking the right support as a center of moments we find the value of R_1 as 4.3875. The sum of R_1 and R_2 is, of course, equal to the total wind load 7.2 tons.

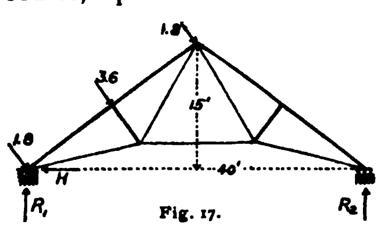

Fig. 17.

When one end of the truss is free and the wind blows on the fixed side, as in Fig. 17, the reaction R_2 at the free end must be vertical. The reaction at the fixed end is inclined, but

it will be convenient to resolve it into a vertical component R_1 and a horizontal component H. To find H we state the condition that the algebraic sum of the horizontal components of the exterior forces must be zero, which gives

$$(1.8 + 3.6 + 1.8) \sin \theta - H = 0, \text{ whence } H = 4.32.$$

To find R_2 take the center of moments at the left support, then

$$R_2 \times 40 - 3.6 \times 12.5 - 1.8 \times 25 = 0, \text{ whence } R_2 = 2.25.$$

R_2 may also be found by regarding the whole wind load, 7.2 tons, as concentrated at the middle of the rafter and resolving it into vertical and horizontal components, 5.76 and 4.32 respectively; then

$$R_2 \times 40 - 5.76 \times 10 - 4.32 \times 7.5 = 0, \text{ whence } R_2 = 2.25.$$

To find R_1, take the center of moments at the right support; then

$$R_1 \times 40 - 5.76 \times 30 + 4.32 + 7.5 = 0, \text{ whence } R_1 = 3.51.$$

As a check $R_1 + R_2$ should equal the vertical wind load component, 5.76 tons.

When one end of the truss is free and the wind blows on the free side, as in Fig. 18, the reaction of the fixed end may be also represented by its vertical and horizontal components, while at the free end the reaction is vertical only. By the use of the fundamental conditions of equilibrium, we find in the same manner as above, $H = 4.32$, $R_1 = 2.25$ and $R_2 = 3.51$. This illustration shows that for the same truss the values of R_1 and R_2 interchange, when the wind changes from one side of the roof to the other, and that H reverses its direction.

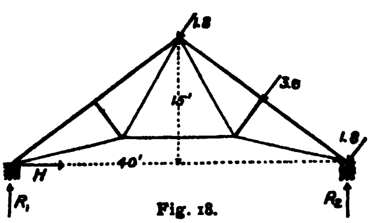

Fig. 18.

For a truss with broken upper chord, like Fig. 19, the same general principles apply. If P_1 and P_2 be the normal wind loads on the two bays and θ_1 and θ_2 the angles which they make with the vertical, we have for the horizontal reaction,

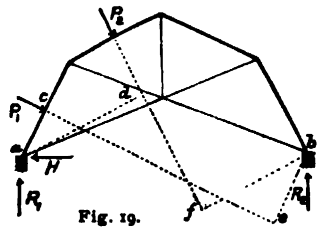

Fig. 19.

$$P_1 \sin\theta + P_2 \sin\theta_2 - H = 0.$$

For the vertical reactions, we have by moments,

$$R_1 \times ba - P_1 \times be - P_2 \times bf = 0,$$

$$R_2 \times ab - P_1 \times ac - P_2 \times ad = 0,$$

and the sum $R_1 + R_2$ must equal $P_1 \cos\theta_1 + P_2 \cos\theta_2$.

Prob. 23. A Fink truss, like Fig. 17, has its span 80 feet, rise 17 feet, total normal wind load, 4.38 tons. Find the reactions for wind on the fixed side.

Prob. 24. For the same truss find the reactions due to wind on the free side.

Prob. 25. A truss, like Fig. 1, has the span 40 feet, rise 12 feet, distance between trusses 12 feet. Find the wind load, and the reactions when both ends are fixed.

ART. 15. WIND STRESSES IN TRUSSES WITH FIXED ENDS.

The stresses caused by the wind may now be computed by the methods of Arts. 5 and 7. As the wind load is unsymmetrical to the truss, the stresses in the corresponding members on the right side are different from those on the left side, and hence the stresses must be found throughout the entire truss.

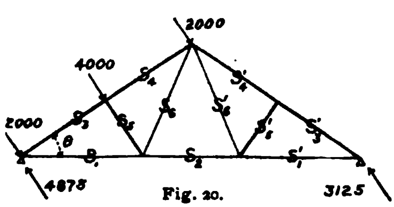

Fig. 20.

As an example, take the truss in Fig. 20, where the span is 48 feet, rise 18 feet, and wind loads and reactions

as shown, both ends being fixed. From the given rise and span the lengths of S_3 and S_4 are found to be 15 feet, of S_1 and S_6, 18.75 feet, and of S_5, 11.25 feet; also, $\cos\theta = 0.8$ and $\sin\theta = 0.6$. Then for the left hand part of the truss, we find

$$2\,875 \times 15 - S_1 \times 9 = 0, \qquad S_1 = +4\,790,$$
$$2\,875 \times 30 - 4\,000 \times 15 - S_2 \times 18 = 0, \qquad S_2 = +1\,460,$$
$$2\,875 \times 15 + S_3 \times 11.25 = 0, \qquad S_3 = S_4 = 3\,830,$$
$$-4\,000 \times 15 - S_5 \times 15 = 0, \qquad S_5 = -4\,000,$$
$$-4\,000 \times 15 + S_6 \times 18 = 0, \qquad S_6 = \times 3\,333.$$

For the right hand part of the truss it will be most convenient to resolve the reaction 3 125 into the horizontal and vertical components, 1 875 and 2 500 respectively, and in stating the equation for any piece, pass a cutting plane and consider the unknown stresses, as in equilibrium with the forces on the right hand side of the section. Thus,

$$2\,500 \times 12 - 1\,875 \times 9 - S'_1 \times 9 = 0, \qquad S'_1 = +1\,460,$$
$$2\,500 \times 24 - 1\,875 \times 18 - S'_2 \times 18 = 0, \qquad S'_2 = +1\,460,$$
$$2\,500 \times 1\,875 + S'_3 \times 11.25 = 0, \qquad S'_3 = -4\,170,$$
$$S'_5 = 0, \qquad S'_6 = 0, \qquad S'_4 = -4\,170.$$

Prob. 26. Find all the wind stresses for the truss with fixed ends, shown in Fig. 16, the rise of the tie being 2 feet.

ART. 16. WIND STRESSES IN TRUSSES WITH ONE END FIXED AND THE OTHER FREE.

For these trusses the stresses are to be computed for all members, taking the wind on one side of the roof, and then again for all members taking the wind on the other side.

Take the truss in Fig. 21, whose span is 60 feet, rise 12 feet, rafter divided into three equal parts, struts normal to rafter, distance between trusses 13 feet 9 inches. By Art. 13, the normal wind load per square foot of roof surface is 19.9 pounds, which gives a total wind load of 8 850 pounds, subdivided into apex loads as shown. For wind on the fixed side, the reactions are found by Art. 14. Finally by the methods of Art. 5 and Art. 7, the stresses due to these loads are computed and marked on the diagram.

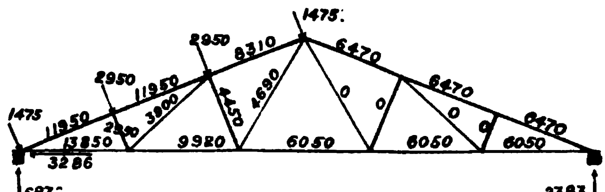

Fig. 21.

In this particular case the method of moments will be found most convenient for all members except the inclined ties, and it will be often best to state the equation including the applied force on the right of the section rather than on the left. Thus, for the second panel of the lower chord the equation for forces on the left is

$$5\ 832 \times 20 + 3\ 286 \times 8 - 1\ 475 \times 21.54 - 2\ 950 \times 10.77 + S \times 8 = 0,$$

while for the forces on the right, it is

$$2\ 383 \times 40 - 1\ 475 \times 10.77 - S \times 8 = 0,$$

from both of which we find, $S = +\ 9\ 920$ pounds.

When the wind blows upon the free side of the roof the stresses are materially different, as shown by the comparison of the values in Fig. 21 with those in Fig. 22. In the first case the tendency of the wind is to flatten the roof, in the second to double it up.

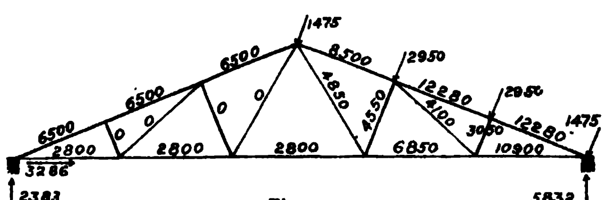

Fig. 22.

Prob. 27. Find the stresses for the truss in Fig. 21, caused by the given wind loads.

Prob. 28. Find the stresses for the truss in Fig. 22, caused by the given wind loads.

Prob. 29. Prove that there are no wind stresses in the bracing on the unloaded side, when both upper and lower chords are straight from the support to the center.

ART. 17. FINAL MAXIMUM AND MINIMUM STRESSES.

The stresses caused by the dead load always exist, and these are increased (or sometimes diminished) by the stresses due to wind and snow. In order to design a member for the range of stress (Mechanics of Materials, Art. 81), it is necessary to know the maximum and minimum stresses due to combination of the dead load with the other loads. The word maximum will here be used as meaning the greatest tensile or greatest compressive stress, and the word minimum as meaning the least stress of the same kind.

Let the data for the truss, in Fig. 23, be as follows: Span = 60 feet, rise of upper chord = 13 feet, rise of lower chord = 2 feet, rafter divided into three equal parts, struts vertical, dead load of truss and roof covering = 12 pounds per square foot of roof surface, distance apart of trusses = 15 feet, snow load per horizontal square foot = 15 pounds, wind load per vertical square foot = 40 pounds, one end of truss on rollers.

From these data the dead load stresses, snow load stresses, and stresses due to wind on each side are computed by the methods of

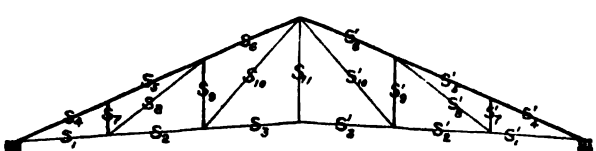

Fig. 23.

the preceding Articles and tabulated, in short tons, as follows: For the lower chord,

	S_1	S'_1	S_2	S'_2	S_3	S'_3
Dead load stresses	+ 6.7	+ 6.7	+ 5.4	+ 5.4	+ 4.0	+ 4.0
Snow load stresses	+ 7.7	+ 7.7	+ 6.2	+ 6.2	+ 4.6	+ 4.6
Wind on fixed side	+ 9.0	+ 3.9	+ 6.4	+ 3.9	+ 3.9	+ 3.9
Wind on free side	+ 1.3	+ 6.4	+ 1.3	+ 3.9	+ 1.3	+ 0.2
Maximum stresses	+ 23.4	+ 20.8	+ 18.0	+ 15.5	+ 12.5	+ 12.5
Minimum stresses	+ 6.7	+ 6.7	+ 5.4	+ 5.4	+ 4.0	+ 4.0

Here, as the stresses due to all the loads are positive, the maximum stress for each member is found by adding the separate stresses, remembering that the wind can only blow upon one side of the roof at the same time, and the minimum stress in each case is that caused by the dead load.

In the same manner we find for the upper chord,

	S_4	S'_4	S_5	S'_5	S_6	S'_6
Dead load stresses	− 7.3	− 7.3	− 7.3	− 7.3	− 5.8	− 5.8
Snow load stresses	− 8.4	− 8.4	− 8.4	− 8.4	− 6.7	− 6.7
Wind on fixed side	− 7.9	− 4.2	− 8.6	− 4.2	− 6.6	− 4.2
Wind on free side	− 3.6	− 7.4	− 3.6	− 8.1	− 3.6	− 6.0
Maximum stresses	− 23.6	− 23.1	− 24.3	− 23.8	− 19.1	− 18.5
Minimum stresses	− 7.3	− 7.3	− 7.3	− 7.3	− 5.8	− 5.8

and for the braces,

	S_7	S'_7	S_8	S'_8	S_9	S'_9	S_{10}	S'_{10}	S_{11}
Dead load stresses	−1.0	−1.0	+1.7	+1.7	−1.5	−1.5	+2.0	+2.0	+0.5
Snow load stresses	−1.1	−1.1	+2.0	+2.0	−1.7	−1.7	+2.3	+2.3	+0.6
Wind on fixed side	−1.9	0	+3.3	0	−2.8	0	+3.9	0	+0.5
Wind on free side	0	−1.9	0	+3.3	0	−2.8	0	+3.9	+0.1
Maximum stresses	−4.0	−4.0	+7.0	+7.0	−6.0	+6.0	+8.2	+8.2	+1.6
Minimum stresses	−1.0	−1.0	+1.7	+1.7	−1.5	−1.5	+2.0	+2.0	+0.5

For this truss the maximum stresses for corresponding members on the fixed and free sides differ but little, and in practice they would be built of the same size, but a truss with curved upper chord and of large span may often have a great variation, or even reversal of stress, in some of the corresponding members.

It should be noted that some authors suppose in finding the maximum stresses that both wind and snow do not act at the same time. Under this supposition the final stresses are always less than by the method above followed; thus, the maximum stresses for S_7 and S_{11} would be -2.9 and $+1.1$ tons respectively.

Prob. 30. Let the span of a wooden truss, like Fig. 17, be 40 feet, rise of peak 15 feet, rise of tie 2 feet, distance apart of trusses 13 feet, one end on rollers, dead and snow loads as in Art. 2. Compute the maximum and minimum stresses in all members.

ART. 18. CRESCENT ROOFS.

It will have been observed when the upper chords are broken, as in Figs. 3 and 12, that the computation of the stresses becomes laborious, on account of the difficulty of finding the lever arms or the sines or cosines of the angles. In practice, the stresses for such trusses are generally found by the methods of Graphic Statics, without the necessity of other computations than the determination of apex loads and reactions. But in all such cases it is usual to compute one or more of the simpler stresses in order to check the graphical work. The chord members are usually straight between adjacent apex points.

Prob. 31. A wrought iron truss of the dimensions given in Fig. 12, has a roof covering weighing twelve pounds per square foot of roof surface, trusses twelve feet apart, snow load 15 pounds and wind load 40 pounds on horizontal and vertical surfaces respectively, one end of the truss on rollers. Find the maximum and minimum stresses in the horizontal tie rod.

ART. 19. PURLINS.

A 'purlin' is a beam placed longitudinally between the trusses, and upon which the roof covering rests, either directly or by means of rafters. The simplest purlin is a wooden beam; next come wrought iron T or I beams; and then follow iron trussed beams.

The simple purlins are investigated exactly like beams. (Mechanics of Materials, Chaps. III and IV.) Let its length be l, which is the distance between the roof trusses; the uniform load upon it be W, which includes its own weight, the roof covering, the snow, and if necessary the wind. For all cases the external bending moment M equals the internal resisting moment $\frac{SI}{c}$. For a beam supported at ends $M = \frac{1}{8}Wl$, but for fixed ends $M = \frac{1}{12}Wl$; usually M is between these values. We have then

$$S = \frac{Wlc}{8I} \qquad \text{or} \qquad S = \frac{Wlc}{12I}$$

as the limiting values of the greatest unit fiber stress S, and by the use of this we may find S and hence the factor of safety of an existing purlin, or having assumed a proper value of S we may design a purlin for a given length and load.

A purlin or beam may be 'trussed' as in Fig. 24. The effect of this is to cause ab to be in compression and adb in tension. It is indeed a small truss, the loads being applied at c, and is computed by the methods of the preceding Articles. The load at c should include the weight of the purlin itself, of the roofing and the snow and wind.

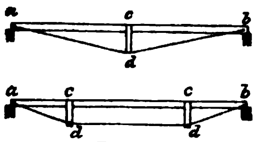

Fig. 24.

If, in the second sketch, $ac = 4$ feet, $cc = 4$ feet and $cd = 2$ feet, and each load at c be 2 000 pounds, we find for the stress in the upper chord 4 000 pounds compression, and in dd also 4 000 pounds tension, while cd has 2 000 pounds compression, and ad has 4 450 pounds tension.

Prob. 32. A roof with a slope of 30° has its trusses 12 feet apart. The purlins are also 12 feet apart. The weight of purlins and roof covering is 12 pounds per square foot of roof surface, and the snow is 15 pounds per square foot of horizontal area; wind not considered. The purlins are I beams fixed at ends, depth 4 inches, width of flanges 2.5 inches, thickness of flanges and web 0.32 inches. Find the greatest unit-stress S, and the degree of security of the purlin.

Prob. 33. For the same data it is required to design a wrought iron trussed purlin of the first form in Fig. 24 and to find the dimensions of all its pieces so that the stress per square inch may be 7 000 pounds.

ART. 20. FLEXURAL STRESSES IN MEMBERS.

The maximum and minimum stresses found by the methods of the preceding Article are those upon a skeleton truss; that is upon members without weight. But the members themselves have weight, and sometimes loads are placed on the lower chord, or purlins attached to the upper chord between the apex points. The effect of these loads is to cause flexural stresses, increasing the longitudinal stress on one side of the member and decreasing it upon the other. (See Chap. VII, Mechanics of Materials.)

The following example will indicate the method of investigating the flexural effect caused by purlins. Let AB be a portion of the upper chord or main rafter between two apex points A and B; its length is 14 feet, size 4 × 6 inches, and at its middle point a purlin brings a load of 840 pounds. The rafter is inclined 30° to the horizontal and the compressive stress upon it is 16 000 pounds. It is required to find the greatest unit compressive stress upon the upper fiber at the middle of the rafter, due to the direct compression and the purlin load.

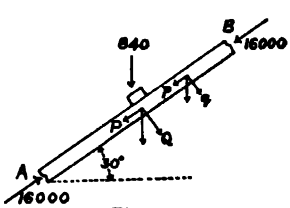

Fig. 25.

For the direct compressive stress we have from GORDON'S formula for columns

$$S_1 = \frac{16\,000}{4 \times 6}\left(1 + \frac{1}{3\,000} \cdot \frac{14^2 \times 12^2}{3}\right) = 2\,760 \text{ lbs. per sq. inch.}$$

For the flexural stresses the vertical load may be decomposed into components parallel and normal to the rafter. The parallel component P is 420 pounds and the normal one Q is 727 pounds. The effect of P is a direct compression, and the unit stress due to it is

$$S_2 = \frac{420}{4 \times 6} = 17 \text{ pounds per square inch.}$$

The effect of Q is a flexural stress, and regarding the beam as fixed we have

$$S_3 = \frac{Mc}{I} = \frac{727 \times 14 \times 12 \times 3 \times 12}{8 \times 4 \times 6 \times 6 \times 6} = 636 \text{ lbs. per sq. inch.}$$

The total unit compressive stress on the upper fiber hence is

$$S = 2\,760 + 17 + 636 = 3\,413 \text{ pounds per square inch,}$$

which is too great for a wooden member, since the factor of safety is only about 2.5.

The effect of the flexure caused by the weight of the members themselves is usually not considered in actual design as it is generally small compared to the total stress. In the above case, taking the weight of the beam at 40 pounds per cubic foot, the direct compression due to the weight is 1 pound per square inch, and that due to the flexure 48 pounds per square inch.

Prob. 34. A wooden upper chord, as in Fig. 25, has a direct compression of 20 000 pounds, and is loaded by a purlin at the middle with 750 pounds. Its length is 16 feet, its size $4 \times d$ inches, and its inclination 45°. Find its depth so that the greatest fiber stress at the middle may be about 800 pounds per square inch.

Prob. 35. A light 4-inch I beam which serves as part of the upper chord is 12 feet long, 30° inclination, and is supported at ends. The compression on it is 18 750 pounds, and it carries three purlins 4 feet apart, each with its load weighing 250 pounds. Find the greatest fiber stress at the middle of the beam.

ART. 21. INVESTIGATION OF ROOF TRUSSES.

To investigate an existing roof truss the following steps are necessary:

(*a*) Measure all the pieces, and ascertain the quality of the materials.

(*b*) Compute all the cross-sections, and the weight of the structure.

(*c*) Assume the proper snow and wind loads.

(*d*) Compute the maximum and minimum stresses in all the members.

(*e*) Find the greatest unit-stresses in all members and connections.

A careful consideration of these operations will show that the investigation of a roof truss is a complex problem requiring great skill, judgment and experience, and that the computation of the stresses is the least difficult part. In the examination of the structure particular attention must be given to the joints and connections, since it often happens that these are weaker than the main members. In a wooden truss many joints have parts subject to shearing, which need careful investigation on account of the slight resistance of timber to this stress. In iron trusses the riveted joints must be tested for the bearing compression as well as for shear and tension, while the pins are to be computed for bending as well as for shearing. In any important case it will be best to make a working drawing showing all details, since thus the data as to dimensions are most clearly presented.

Prob. 36. A roof truss, like Fig. 11, has 100 feet span and 20 feet rise. The upper and lower chords are timbers, the former

10 × 10 inches and the latter 6 × 10 inches, the maximum stresses upon which, due to dead, snow and wind loads, are 5 200 and 4 900 pounds. Find the unit-stresses.

ART. 22. DESIGN OF ROOF TRUSSES.

In making a design for a proposed roof its span is generally given, and also certain limits regarding its height and style. The following are then the steps of procedure:

(*a*) Design the roof covering, purlins, etc., and find their weights.
(*b*) Make a skeleton outline of the proposed truss.
(*c*) Assume the proper snow and wind loads.
(*d*) Compute the maximum stress in all members.
(*e*) Assume the proper working unit-stresses for the materials.
(*f*) Design the sections and the connections.
(*g*) Make drawings, compute weights and estimate the cost.

It will be seen that the process of design is far more difficult than that of investigation. The computation of stresses is the least part of the problem, being merely a mathematical exercise, whose solution is easy when the data are known. But in the determination of the data and in the execution of the design, great ingenuity, judgment and experience are required in order to produce a safe and economical structure. The roof is to be built so that all parts of it possess the proper degrees of security, and so that its cost shall be the least possible. Often several designs must be investigated to secure this result.

In Art. 17 the minimum stress in each member, as well as the maximum stress, is determined in order to ascertain the range of stress. It should be noted, however, that repeated stresses in roof truss members occur at wide intervals of time, and hence are not so injurious as in bridge trusses. In designing roof truss members the maximum stress alone is usually employed, the minimum stress being disregarded, or if regarded at all, a

slightly lower working unit-stress is used for those members subject to great ranges.

The formulas for the weights of trusses, stated in Art. 2, give only rough approximate values, and should never be used when the actual weights of trusses similar in style to those of the proposed design can be obtained. On the completion of a design the computed weight of the truss should be compared with the approximate assumed weight, and if a difference so great as six or eight per cent. be found, it may be necessary to repeat the computations and revise the entire design, so that the assumed and computed weights shall agree.

If specifications as to roof covering, loads, properties of materials, etc., be given in advance to the designer, as is generally the case, his task is rendered easier and his responsibility lighter. Such specifications should always be carefully prepared and made a part of the contract between the buyer and the builder of the roof.

Prob. 37. Design the main outlines of a plan for a roof over a building 80 feet wide and 250 feet long.

CHAPTER II.

HIGHWAY BRIDGE TRUSSES.

ART. 23. DEFINITIONS.

A bridge truss, like a roof truss, consists of members so arranged that each is subject only to stress in the direction of its length. For the same reasons as given in Art. 1, the elementary figures should be triangles, and the loads be applied only at the apex or panel points.

A 'simple truss' is one supported at the two ends, while a 'continuous truss' is supported at more than two points. Only simple trusses will be considered in this Chapter and the next.

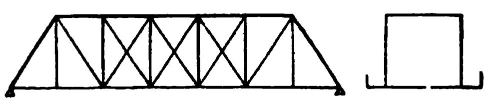

Fig. 26.

A bridge truss consists of the 'upper chord,' the 'lower chord' and the 'braces' or 'web members,' whose functions are similar to the corresponding parts of roof trusses. The upper chord is in compression and the lower in tension, like the top and bottom fibers of a simple beam, while the members composing the bracing (or webbing) are some tensile and some compressive.

A bridge usually consists of two trusses connected by a floor attached to the panel points of either upper or lower chords, while the other chords are united by 'lateral bracing.' When the floor is placed upon the upper chords it is termed a 'deck bridge,' and when upon the lower chords a 'through bridge.' When a through bridge is so low that lateral bracing cannot be used between the upper chords, the trusses are called 'pony trusses.'

The bridge floor consists of 'floor beams,' which run at right angles to the chords, and are connected to them at the apex or panel points, 'stringers' which rest upon the floor beams and are parallel to the chords, and the planks, which rest upon the stringers and support the load. The roadway is that part of the floor between the two trusses, while the sidewalks, if any, are placed outside of the trusses.

The general principles of Arts. 4—7 enable the stresses in bridge trusses due to given loads to be computed.

Prob. 38. A floor beam for a highway bridge is 16 feet long and is to carry a uniformly distributed load of 18 600 pounds. Find its depth, taking the width as 12 inches, so that the maximum fiber stress may be 800 pounds per square inch.

Prob. 39. Find the stress in the end panel of the lower chord in Fig. 26, due to a load of 6 400 pounds placed at each apex point of the lower chord.

ART. 24. DEAD LOADS.

The dead load of a highway bridge consists o. the weight of the floor, lateral bracing, trusses and all the pieces that connect and stiffen them. This weight depends upon the style of the bridge, upon its width and span, upon the live loads and unit-stresses adopted, so that it is subject to much variation in particular cases. The following values are hence to be regarded as approximate, and only useful when more precise information cannot be obtained.

The 'floor system' consists of planks resting upon stringers, and the latter resting upon the floor beams. The planks weigh from 6 to 16 pounds per square foot of floor surface, depending upon the thickness and kind of lumber, say 10 pounds for a mean value. The stringers vary in weight according to their distance apart and their span, while the floor beams vary in weight, depending upon the width of the bridge, the lengths of the panels, the live loads, and other circumstances. The total floor system may weigh from 15 to 25 pounds per square foot of floor.

The lateral bracing which connects the trusses between the two upper chords and between the two lower chords, resists the stresses caused by the wind. Its weight per square foot of floor surface increases from about 2 pounds at 50 feet span, to about 5 pounds at 300 feet span.

The trusses increase in weight with the width and span of the bridge.

The total dead load, or own weight, of a highway bridge may be roughly expressed by the following empirical formula:

$$w = 140 + 12b + 0.2bl - 0.4l, \qquad (2)$$

in which l is the span in feet, b the width of the bridge in feet (including sidewalks, if any), and w is the dead load in pounds per linear foot. This formula gives weights closely agreeing with those of class A in WADDELL'S 'Highway Bridges,' which are somewhat heavier than the actual weights of most country bridges.

The width in the clear between the trusses of a highway bridge is rarely less than 16 feet, and usually not greater than 24 feet, except in large cities. The sidewalks are generally outside of the trusses, supported upon the projecting floor beams.

Prob. 40. If a highway bridge cost 3.5 cents per pound, find the approximate cost of one 24 feet wide and 100 feet span; also the cost of one 24 feet wide and 200 feet span.

Prob. 41. A distance of 720 feet between two abutments is to be spanned by highway bridges 23 feet wide. If each pier cost $12 000 and the bridges cost 3.5 cents per pound, compute the approximate cost of 1 pier and 2 spans, of 2 piers and 3 spans, and of 3 piers and 4 spans, in order to ascertain the most economical plan.

ART. 25. KINDS OF TRUSSES.

A few of the most important and simple kinds of bridge trusses will now be explained briefly.

Fig. 27 shows a skeleton diagram, and also an outline of the king-post truss as formerly built for country highway bridges of short span. A load at the center is carried up the vertical tie and then by the two inclined struts to the abutments. This tie is now generally made of a wrought iron rod.

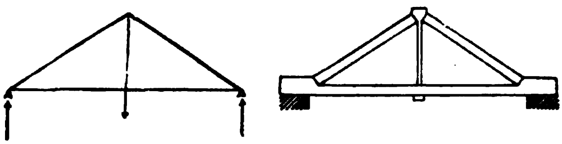

Fig. 27.

The queen-post truss, sometimes called a quadrangular truss, has two vertical ties which carry the panel loads to the upper chord, whence they are brought to the abutments by the inclined struts.

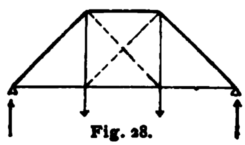

Fig. 28.

The Burr truss, Fig. 29, has its vertical members in tension and the inclined ones in compression. This may be regarded as an extension of the king and queen post trusses; it is an old form, built wholly in wood and now no longer used.

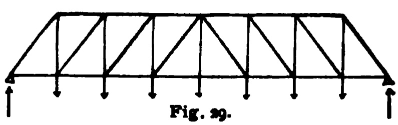

Fig. 29.

The Howe truss is like the Burr in principle, except that diagonal counter-struts, here represented by broken lines, are introduced to prevent the distortion caused by the live load. The vertical ties are wrought iron and all the other members are wood. This truss has been extensively built on account of facility of construction.

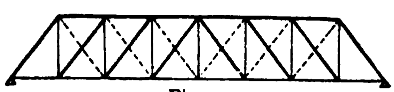

Fig. 30.

The Warren truss, Fig. 31, has all its web members inclined at equal angles, some of them being in tension and some in compression. This form, like the last, is represented as a through truss, but they are also often used for deck bridges. The Warren truss is generally built all in iron.

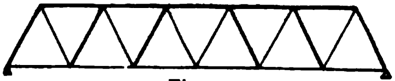

Fig 31.

The Pratt truss is a favorite type which has the verticals in compression and the diagonals in tension. Fig. 32 shows both the deck and the through form, the counter-ties being omitted. In the through truss the two verticals nearest the ends are in tension, while the inclined end members are of course in compression. This truss was formerly built with the diagonal ties of wrought iron and the other parts of wood; now all members are usually iron.

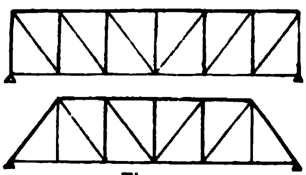

Fig. 32.

A 'combination truss' is one in which some of the members are iron and others wood.

The details of construction, that is, the arrangement of connections and joints, the style and proportions of struts, the kind of end supports, etc., are quite different in trusses of different manufacturers. The student should, however, embrace every opportunity to become familiar with these details, either by the actual inspection of bridges, or by the careful study of working drawings. As every technical school has charts, photographs and drawings illustrating these details, it is not thought necessary to dwell upon them here.

In Europe trusses are generally built with riveted joints, such forms as the Burr, Howe, and other combination trusses being unknown. In this country, however, iron trusses of more than 75 feet span are usually built with pin connections, although there are now many indications that riveted bridges are preferable for all spans less than 150 feet.

Prob. 42. A king-post truss has 18 feet span and 9 feet rise. Compute the stresses in all the members due to a load of 14 000 pounds at the middle.

Prob. 43. A queen-post truss has 30 feet span and 10 feet depth, the three panels of the lower chord being equal. Find the stresses in all members due to two loads, each 12 000 pounds, at the panel points.

ART. 26. STRESSES IN WEB MEMBERS.

There are certain members of the webbing, called counter-struts or counter-ties, which are not strained by the dead load and which only come into action when the live load crosses the bridge. In making a skeleton diagram of a truss in order to compute the stresses due to dead load, these counters should be omitted.

> For trusses with horizontal chords the stress in any web member is equal to the vertical shear multiplied by the secant of the angle which the member makes with the vertical.

This important rule is readily deduced from the principle of resolution of forces explained in Art. 7, or from the second condition of static equilibrium. Thus, in Fig. 33, let a section be drawn cutting three members; then the three unknown stresses are in equilibrium with the exterior forces on the left of the section, and hence the algebraic sum of the vertical components of all these forces equals zero. Therefore, if θ be the angle between the web member and the vertical, we have

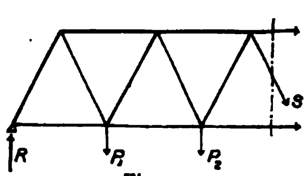

Fig. 33.

$$R - P_1 - P_2 - S \cos \theta = 0.$$

But $R - P_1 - P_2$ is the vertical shear V for the given section

(Mechanics of Materials, Art. 16); hence we have,

$$V - S\cos\theta = 0, \quad \text{or} \quad S = V\sec\theta,$$

which proves the rule for trusses with horizontal chords.

For the particular diagonal shown in Fig. 33, the stress S will be tension, provided that V be positive, but for the next preceding diagonal we have $V + S\cos\theta = 0$, or $S = -V\sec\theta$, and its stress will be compression if V be positive. Hence, we determine the kind of stress by regarding the direction of the arrow and the sign of the shear. For the dead load the shear is always positive if the section be on the left of the middle of the bridge.

For example, let the through Howe truss, in Fig. 34, have 8 panels, each 18 feet long, and 27 feet deep. The dead load per linear foot per truss is 444 pounds, which gives an apex panel load of 8 000 pounds or 4 short tons. Each reaction is 14 tons. The secant of the angle between the vertical and a diagonal is $\frac{\sqrt{18^2 + 27^2}}{27}$ or 1.202.

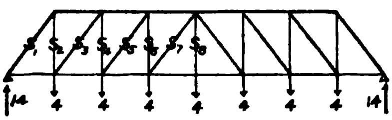

Fig. 34.

The secant for the vertical ties is 1. For the diagonals we now find,

$$S_1 = -14 \times 1.202 = -16.8,$$
$$S_3 = -(14 - 4) \times 1.202 = -12.0,$$
$$S_5 = -(14 - 8) \times 1.202 = -7.2,$$
$$S_7 = -(14 - 12) \times 1.202 = -2.4,$$

and for the verticals,

$$S_2 = +14, \qquad S_4 = +10, \qquad S_6 = +6 \text{ tons.}$$

The stress S_8 cannot be found by the above method, since a section cutting it and the upper chord passes through four pieces. But by passing a section cutting S_8 and the two lower chords we have at once $S_8 = +4$ tons.

If the student finds any difficulty in determining the sign of the stress it can always be overcome by drawing a section cutting the piece, marking the arrow, and referring to the fundamental conditions of static equilibrium (Art. 4).

Prob. 44. A deck Pratt truss has 8 panels, each 15 feet long, and its depth is 20 feet. Find the stresses in all the web members due to a dead load of 450 pounds per linear foot per truss.

Prob. 45. A through Warren truss has 11 panels, each 8 feet long and its depth is 8 feet. Find stresses in all web members due to a dead load of 380 pounds per linear foot per truss.

Prob. 46. Find the web stresses for a deck Warren truss with the same dimensions and loads as in the last problem.

ART. 27. STRESSES IN CHORDS.

For finding the chord stresses either the method of moments (Art. 5) or the method of resolution of forces (Art. 7), may be used. In either case a skeleton diagram should be made, the counters, if any, being omitted.

By the method of moments we proceed as follows:

> Pass a section cutting the given chord member, take the center of moments at the intersection of the other two pieces, and state the equation of moments between the unknown stress and the exterior forces on one side of the section.

For example, let the through Warren truss, in Fig. 35, have 6 panels, each 10 feet long, its depth being also 10 feet. If the dead load

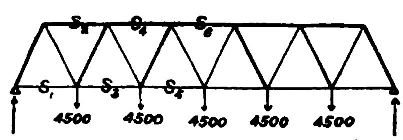

Fig. 35.

per linear foot is 450 pounds, each panel load is 4 500 pounds and each reaction 11 250 pounds. Then for the upper chord stresses,

$11\,250 \times 10 + S_2 \times 10 = 0, \qquad S_2 = -11\,250,$
$11\,250 \times 20 - 4\,500 \times 10 + S_4 \times 10 = 0, \qquad S_4 = -18\,000,$
$11\,250 \times 30 - 4\,500 \times 20 - 4\,500 \times 10 + S_6 \times 10 = 0,$
$S_6 = -20\,250,$

and for the lower chord stresses,

$11\,250 \times 5 - S_1 \times 10 = 0, \qquad S_1 = +5\,625,$
$11\,250 \times 15 - 4\,500 \times 5 - S_3 \times 10 = 0, \qquad S_3 = +14\,625,$
$11\,250 \times 25 - 4\,500 \times 15 - 4\,500 \times 5 - S_5 \times 10 = 0,$
$S_5 = +19\,125.$

By the method of resolution of forces, or 'method of chord increments,' as it is sometimes called, the following is the process for horizontal chords:

> Pass a section cutting the given chord member and all the braces on the left, find the shears for those braces, multiply each shear by the tangent of its angle with the vertical, and take the sum of the products for the chord stress.

To prove this rule, let S', S'', etc., be the stresses in the braces cut by the section, V', V'', etc., the shears for those braces, and θ', θ'', etc., the angles which they make with the vertical. Then from the first condition of equilibrium the sum of the horizontal components is zero, or

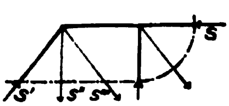

Fig. 36.

$$S = S' \sin \theta' + S'' \sin \theta'' + S''' \sin \theta''' + \text{etc.}$$

But, as shown in the last Article, $S' = V' \sec \theta'$, $S'' = V'' \sec \theta''$, etc., hence

$$S = V' \tan \theta' + V'' \tan \theta'' + V''' \tan \theta''' + \text{etc},$$

which proves the proposition as stated.

For example, take the Warren truss, in Fig. 35, where the

braces are all equally inclined, and tan $\theta = \frac{5}{10} = 0.5$. Then for the lower chord

$S_1 = 12\,250 \times 0.5 = 5\,625,$
$S_3 = (11\,250 + 11\,250 + 6\,750) \times 0.5 = 14\,625,$
$S_5 = (11\,250 + 11\,250 + 6\,750 + 6\,750 + 2\,250) \times 0.5 = 19\,125,$

and for the upper chord,

$S_2 = (11\,250 + 11\,250) \times 0.5 = 11\,250,$
$S_4 = (2 \times 11\,250 + 2 \times 6\,750) \times 0.5 = 18\,000,$
$S_5 = (2 \times 11\,250 + 2 \times 6\,750 \times 2 \times 2\,250) \times 0.5 = 20\,250.$

The two methods give the same results, as should be the case; hence one may be used to check the other. The method of moments is general and applies to any form of truss, but the method of increments is only valid when the chords are horizontal.

Prob. 47. Find all the chord stresses for the truss of Prob. 45.

Prob. 48. Find all the chord stresses for the truss of Prob. 46

Prob. 49. A through Pratt truss has 9 panels, each 15 feet long, and its depth is 20 feet. Find the stresses in all the chord members due to a load of 450 pounds per linear foot per truss.

ART. 28. DEAD LOAD STRESSES.

In the examples of the two preceding Articles the entire dead load has been regarded as concentrated upon the upper chord in deck bridges and upon the lower chord in through bridges. This is the usual plan; but sometimes it is specified that the dead load shall be divided between the chords, each chord taking one-half of the weight of the trusses and lateral bracing, and the floor load being supported entirely by the chord upon which it rests.

To illustrate this method we take a through Pratt truss of 9 panels, each 18 feet long, the depth being 24 feet, the weight of the floor system 433 pounds and of the trusses and lateral bracing 600 pounds per linear foot of bridge. This gives for the panel load on the lower chord

$$\tfrac{1}{2}(433 + 300) \times 18 = 6\,597 \text{ pounds} = 3.3 \text{ short tons,}$$

and for the panel load on the upper chord

$$\tfrac{1}{2} \times 300 \times 18 = 2\,700 \text{ pounds} = 1.35 \text{ short tons.}$$

Each reaction is now 18.6 tons, and by the methods of the last two Articles the stresses are computed. For instance, to find the stress in the second vertical from the center pass a plane cutting it and the two chords, then we have

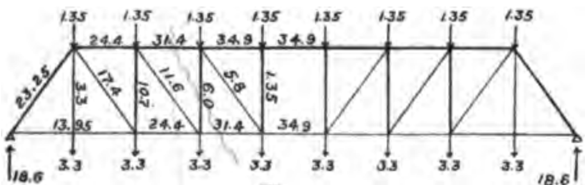

Fig. 37.

$$S = (18.6 - 3 \times 3.3 - 2 \times 1.35) \times 1 = 6.0 \text{ tons compression,}$$

also for the first panel of the upper chord,

$$S \times 24 = 18.6 \times 36 - 4.65 \times 18 \text{ or } S = 24.4 \text{ tons compression.}$$

Thus all the stresses due to the dead load are found and marked upon the diagram.

Prob. 50. A deck Howe truss of 120 feet span has 10 panels and is 18 feet deep. The weight of the floor per linear foot of bridge is 350 pounds and that of the trusses and lateral bracing is 450 pounds. Find the dead load stresses in all members.

ART. 29. LIVE LOADS.

The 'live load' or 'rolling load,' is that which passes over the bridge, like the trains on railroad bridges, or the wagons and foot passengers on highway bridges. This load usually causes heavier

stresses than the dead load, and is also injurious on account of the shocks and repeated stresses produced by it. Highway bridges have been known to fall under the passage of marching troops.

The floor system of highway bridges is to be computed for a densely packed crowd of people, and also for heavily loaded wagons passing over it, the stringers and floor beams being designed by the help of the theory of beams.

The trusses of highway bridges are found to be most highly strained by a crowd of people densely packed upon the roadway and sidewalks. The following are the values usually taken for this live load, all in pounds per square foot of floor surface:

	For City Bridges.	For Country Bridges.
Spans under 50 feet,	100	90
Spans 50 to 125 feet,	90	80
Spans 125 to 200 feet,	80	70
Spans over 200 feet,	70	60

This live load is taken greater for short spans than for long ones, and greater for city than for country bridges, on account of the larger liability to densely packed crowds.

The live load per linear foot of bridge is found by multiplying the clear width of roadway and sidewalks by the given weight per square foot. This load may cover a part or the whole of the bridge, or may move over it, like a train on a railroad bridge. For any given truss member the live load is to be so placed as to produce the largest possible stress.

Prob. 51. A bridge for a country town has its roadway 18 feet wide in the clear, and also two sidewalks, each 5 feet in the clear. The span is 135 feet and there are 9 panels. Find the live panel loads per truss.

Prob. 52. If a highway bridge is 20 feet wide, find approximately the span for which the dead load equals the live load.

ART. 30. CHORD STRESSES DUE TO LIVE LOAD.

The uniform live load as it passes over the bridge causes the stresses in each member of the truss to continually vary. It is therefore important to ascertain the position of the live load which gives the largest possible stress in the member. For any chord member we have the following important theorem:

> The largest stress in any chord member, due to a uniform live load, occurs when the live load covers the entire bridge.

To prove this let Fig. 38 be a truss whose span is l and depth d, and let a section be passed at any distance k from the left support. Now let live loads be placed upon the right of this section producing the reaction R. The chord stress for

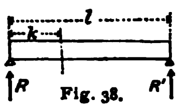

Fig. 38.

the given section then is $S = \frac{Rk}{d}$ which increases with R. Hence, as R increases with the number of loads placed on the right of the section, S also increases with their number. Again, let loads be placed on the left of the section producing the reaction R' at the right end. The chord stress due to these loads then is $S' = \frac{R'(l-k)}{d}$, and this increases with R' or with the number of loads placed on the left. Hence, every load whether on the right or left of the section, increases the chord stress, and therefore the largest chord stress in any member occurs when the live load covers the whole bridge.

The chord stresses due to live load are hence computed in exactly the same manner as those due to dead load, the panel loads being placed at the apex points of that chord which supports the floor.

Prob. 53. A deck Howe truss of 120 feet span has 10 panels and its depth is 17 feet. Compute the chord stresses due to a live load of 900 pounds per linear foot per truss.

ART. 31. MAXIMUM CHORD STRESSES.

The chord stresses due to dead load may be computed as in Arts. 27 and 28, and those due to live load as in Art. 30; the sum of these for any member is then the maximum chord stress due to both dead and live loads. But the same result may be obtained by adding together at first the dead and live panel loads and then computing the stresses. This method is shorter and is hence often employed.

For example, take a through Pratt truss of 11 panels, one-half of which is shown in Fig. 39, the span being 176 feet and the depth 20 feet. The dead panel load at each apex of the upper chord is 2 short tons, and at each apex of the lower chord 4 tons. The live panel load is 11.5 tons. These are placed on the diagram and the reaction found to be 87.5 tons. To find the stresses in the upper chord we now use either of the methods given in Art. 27, and have

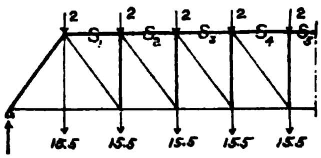

Fig. 39.

$$S_1 = -126,\ S_2 = -168,\ S_3 = -196,\ S_4 = S_5 = -210 \text{ tons.}$$

which are the maximum stresses caused by dead and live loads.

It is seen that for the Howe and Pratt trusses the chord stresses are the same whether the dead load be placed all on one chord or be divided between the two chords. For the Warren truss, however, this is not the case.

Prob. 54. Find the lower chord stresses for the Pratt truss in the above example.

Prob. 55. A through Warren truss of 176 feet span has 11 panels, each 16 feet long, and its depth is 20 feet. The dead load per linear foot of bridge is 500 pounds for the floor system and 1000 pounds for the trusses and lateral bracing. The dead load per linear foot is 2 875 pounds. Find the maximum chord stresses.

ART. 32. VERTICAL SHEARS DUE TO LIVE LOAD.

When the live load crosses the bridge the stresses in the web members vary. The stress S in any such member equals $V \sec \theta$ (Art. 26) and hence we need to find when the vertical shear V is the largest possible. At any point let a section cut the truss, then for this section the following theorem is true for the shears due to the live load:

> The largest positive shear occurs when the live load extends from the section to the right abutment, and the largest negative shear occurs when the live load extends from the section to the left abutment.

To prove this let Fig. 40 be a truss cut at any point by a section, and let loads be placed upon the right of the section producing the reaction R. The shear V for this section then equals R, and R is the greatest when all the panel points between the section and the right abutment are covered by the live load.

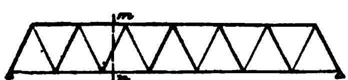

Fig. 40.

Again let loads equal to P be placed on the left of the section causing a reaction R; then the shear is $R - P$, which is negative since R is less than P, and this is numerically increased by every load placed on the left of the section. Therefore, the theorem is proved.

For example, let the live panel load for Fig. 40 be 8 tons. Then to find the largest positive shear for the section *mn* we place the four panel loads on the right, and have

$$V = R = 8\left(\tfrac{1}{7} + \tfrac{2}{7} + \tfrac{3}{7} + \tfrac{4}{7}\right) = +11.43 \text{ tons},$$

and to find the largest negative shear we place the two panel loads on the left, and have

$$V = R - 16 = 8\left(\tfrac{5}{7} + \tfrac{6}{7}\right) - 16 = -3.43 \text{ tons}.$$

The shear for this section due to a load at every panel point is

8 tons, which is the same as the algebraic sum of the two greatest shears just found.

The live load hence produces shears of different kinds when moving in opposite directions, and consequently the dead load stress in any web member is decreased in one case and increased in the other.

Prob. 56. Find the largest positive and negative shears for each panel of a Howe truss of 150 feet span with 10 panels, the apex live load being 7 short tons.

Prob. 57. Find the largest positive and negative shears for a truss of 180 feet span with 9 panels caused by a live load of 1100 pounds per linear foot per truss.

ART. 33. MAXIMUM AND MINIMUM SHEARS.

The shears due to the dead load may be found by Art. 26, and those due to the live load by Art. 32, and their addition will give the resultant maximum and minimum shears. The maximum shear is always positive on the left of the middle of the truss, but the minimum shear may be either positive or negative.

For example, let Fig. 41 be a Howe truss of 8 panels, the dead panel load being 3.6 tons and the live panel load 12 tons. For the dead load shear in the fourth panel we have $V_4 = 12.6 - 3 \times 3.6 = +1.8$ tons. For the largest positive shear the live load covers the four panel points on the right, and

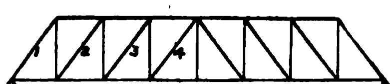

Fig. 41.

$$V_4 = 12\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8}\right) = +15.0 \text{ tons.}$$

For the largest negative shear the live load covers the three panel points on the left, and

$$V_4 = 12\left(\tfrac{5}{8} + \tfrac{6}{8} + \tfrac{7}{8}\right) - 3 \times 12 = -9.0 \text{ tons.}$$

The maximum shear in this panel due to dead and live loads is then 1.8 + 15.0 = + 16.8 tons, and the minimum shear is 1.8 — 9.0 = —7.2 tons. Thus we find the following values for the four panels

	V_1	V_2	V_3	V_4
Dead load shear,	+ 12.6	+ 9.0	+ 5.4	+ 1.8
Live load positive shear,	+ 42.0	+ 31.5	+ 22.5	+ 15.0
Live load negative shear,	0.0	— 1.5	—4.5	— 9.0
Maximum shear,	+ 54.6	+ 40.5	+ 27.9	+ 16.8
Minimum shear,	+ 12.6	+ 7.5	+ 0.9	— 7.2

For this case, then, a negative shear can only occur in the panel nearest the middle, the shears in the other panels always being between the positive values found.

Instead of finding the dead and live load shears separately, the maximum and minimum shears for any panel may be determined by placing the dead and live loads in proper position and then computing their values at one operation. Thus for panel No. 3, to find the maximum shear the dead load covers the whole bridge and the live load is placed at the five panel points on the right, then

$$V_3 = 12.6 + 12\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8}\right) - 2 \times 3.6 = +27.9,$$

and for the minimum shear the live load is on the left, and

$$V_3 = 12.6 + 12\left(\tfrac{6}{8} + \tfrac{7}{8}\right) - 2 \times 3.6 - 2 \times 12 = +0.9.$$

It is often only necessary to compute the minimum shear for those panels where its value becomes negative, as will be seen in the following Articles.

Prob. 58. Find the maximum and minimum shears for a truss of 9 panels, the apex dead and live loads being 4 tons and 15 tons respectively.

ART. 34. WEB STRESSES IN THE WARREN TRUSS.

Let the deck Warren truss, one-half of which is shown in Fig. 42, have 7 panels, each 18 feet long and its depth be 12 feet. Let the dead load per linear foot per truss be 570 pounds, one-third of which is to be taken on the lower chord and two-thirds on the upper chord. Let the live load per linear foot per truss be 1 700 pounds. It is required to find the maximum and minimum stresses in all the web members.

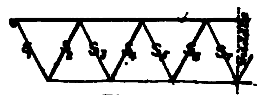
Fig. 42.

The dead panel loads are first found to be 3 420 pounds for the lower chord and 6 840 pounds for the upper chord. A full dead panel load should be taken for the first panel point of the lower chord. The live panel load is 30 600 pounds. The dead load reaction is 32 490 pounds.

The maximum and minimum shears are now to be found for both web members, and these multiplied by sec θ will give the required stresses. The value of sec θ is $\frac{\sqrt{9^2+12^2}}{12} = 1.25$.

For instance, to find the maximum shear in S_6 we pass a section cutting it, place the live load on the right, and have

$$V_6 = 32\,490 + 30\,600\left(\tfrac{1}{7}+\tfrac{2}{7}+\tfrac{3}{7}+\tfrac{4}{7}\right) - 2\times 6\,840 - 3\times 3\,420 = +52\,264.$$

For the minimum shear the live load is on the left of the section, and

$$V_6 = 32\,490 + 30\,600\left(\tfrac{5}{7}+\tfrac{6}{7}\right) - 2\times 6\,840 - 3\times 3\,420 - 3\times 30\,600 = -4\,564.$$

In the same manner for S_5, we pass a section cutting it, and find

$$V_5 = 32\,490 + 30\,600\left(\tfrac{1}{7}+\tfrac{2}{7}+\tfrac{3}{7}+\tfrac{4}{7}\right) - 2\times 6\,840 - 2\times 3\,420 = +55\,684,$$

$$V_5 = 32\,490 + 30\,600\left(\tfrac{5}{7}+\tfrac{6}{7}\right) - 2\times 6\,840 - 2\times 3\,420 - 2\times 30\,600 = -1\,144.$$

For S_4 the minimum shear is positive, and the same is the case for all members preceding S_4.

Regarding the sign of the shear and the direction of the member (Arts. 7 and 26), we now multiply the shear by the secant 1.25 and find the following stresses, + as usual denoting tension and — denoting compression, and the values being given to the nearest hundred pounds.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7
Max.	+155 400	−151 200	+109 800	−105 500	+69 600	−65 300	+34 900
Min.	+40 600	−36 300	+22 300	−18 100	−1 400	+5 700	−30 600

These figures show that the members S_1 and S_3 should be ties to carry tension only, that S_2 and S_4 should be struts to carry compression only, and that S_5, S_6 and S_7 should be members capable of resisting both tension and compression. When a member is so constructed that it will resist both tension and compression, it is said to be 'counter-braced.' This example shows that the diagonals near the middle of a Warren truss need to be counter-braced in order to provide for the stresses due to the live load.

The above values give the range of stress in each member. Thus, for S_3 the stress under dead load is + 27 600 pounds, but when the live load approaches it from the right this is increased to + 109 800, and when from the left, it is decreased to + 22 300 pounds. The piece must hence be designed not only for the maximum stress, but also for the range of stress. It is seen that the members near the middle of the truss have the greatest range of stress.

Prob. 59. A through Warren truss has the same dimensions as that of the above example, and is subject to the same loads Compute the maximum and minimum web stresses.

ART. 35. PANEL COUNTER-BRACES.

A panel of the Howe or Pratt truss is said to be counter-braced when a diagonal is placed therein, crossing the main diagonals, for the purpose of taking the negative shear due to the live load.

In the Howe truss the verticals are iron ties which take only tension, and the diagonals are wooden struts which can take only compression. Under the action of dead load the seven-panel truss, in Fig. 43, needs no braces in the middle panel because the shear is there zero, and on the left of the middle the main braces take the compression due to the positive shears. But under the action of the live load the shears in some of these panels may be negative, and as the main struts cannot take the tension which these would produce, pieces called 'counter-struts' are introduced.

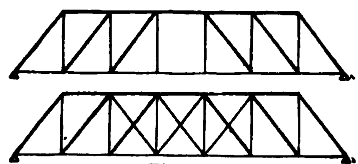

Fig. 43.

A practical way of considering the subject is to regard the panel as distorted by the deflection of the truss under its load. In the first position of Fig. 44, the dead load causes a distortion so that the points a and b are brought nearer together, and hence compression is produced in ab. In the second position the live load causes the points a and b to separate and brings the points c and d nearer together, and as ab cannot take tension, a counter-strut cd must be introduced. This illustration also shows that both diagonals in a panel cannot be strained at the same time.

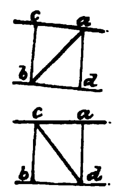

Fig. 44.

The Burr truss, Fig. 29, was a defective one on account of the absence of counter-braces, and was usually stiffened by a wooden arch.

In the Pratt truss the verticals are struts which take only compression, and the diagonals are ties which can take only tension. Accordingly, each panel on the left of the middle where negative shear can occur must have a 'counter-tie,' as also each panel on the right of the middle where positive shear can occur.

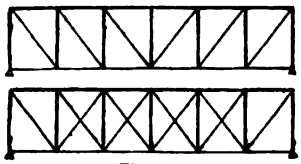

Fig. 45.

To determine the number of panels which require counter-braces is hence easy. Confining our attention to the left hand half of the truss, let the live load come on from the left, and compute the shear in each panel due to both live and dead loads. If this is negative the panel requires a counter-brace, if positive, not. For example, let the dead panel load for the Howe truss, in Fig. 43, be 2 tons, and the live 14 tons, then

$$V_1 = +6 + 0 = +6, \qquad V_2 = +4 + 12 - 14 = +2,$$
$$V_3 = +22 - 28 = -4, \qquad V_4 = 0 + 30 - 42 = -12.$$

Hence, the third and fourth panels require counter-struts, and of course also the fifth. For practical reasons counters are generally placed in more panels than are theoretically necessary.

Prob. 60. Find the number of panels to be counter-braced in Fig. 45, the dead panel load being 2 tons and the live 12 tons.

ART. 36. WEB STRESSES IN HOWE AND PRATT TRUSSES.

The principles now established enable us to find the maximum and minimum stresses in the web members of these trusses. For example, take the through Pratt truss of Art. 28 which has 9 panels, each 18 feet long and 24 feet deep, the dead panel load for the upper chord being 1.35 tons and for the lower chord 3.3 tons. Let the panel live load be 11.7 tons, all of course on the lower chord.

Draw two diagrams of the left hand part of the truss, as in Fig. 46, and for the first suppose the live load passing over the bridge from the right, and in the second, from the left. For the first case we find the largest positive shear for each diagonal, and for the second, the largest negative shear. Thus,

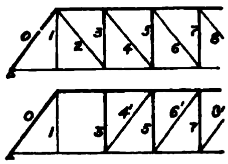

Fig. 46.

	V_0	V_2	V_4	V_6	V_8
Dead load,	+ 18.6	+ 13.95	+ 9.3	+ 4.65	0.0
Live load from right	+ 46.8	+ 36.40	+ 27.3	+ 19.50	+ 13.0
Live load from left	0.0	− 1.30	− 3.9	− 7.80	− 13.0
Maximum	+ 65.4	+ 50.35	+ 36.6	+ 24.15	+ 13.0
Minimum	+ 18.6	+ 12.65	+ 5.4	− 3.15	− 13.0

Multiplying these values by sec θ, which is 1.25, we have the following final stresses for the diagonals:

	S_0	S_2	S_4	S_6	S_8	S'_8	S'_6
Maximum	− 81.8	+ 62.9	+ 45.8	+ 30.2	+ 16.3	+ 16.3	+ 3.9
Minimum	− 23.3	+ 15.8	+ 6.7	0.0	0.0	0.0	0.0

The maximum stresses for the verticals S_3, S_5 and S_7 are the maximum shears found by bringing the live load on from the right, and are − 37.9, − 25.5 and − 14.4 respectively. The minimum stresses are those due to dead load, namely, − 10.7, − 6.0 and − 1.4 tons. The vertical S_1 is evidently always in tension, the maximum being the full panel load 3.3 + 11.7 = 15 tons, and the minimum being the dead panel load 3.3 tons. Some designers would insert the counter-tie S'_4, although none is theoretically required.

Prob. 61. Find the web stresses for the truss of Prob. 50, taking the live load as 1 200 pounds per linear foot per truss.

ART. 37. RANGES OF STRESS.

The preceding Articles show that the chord stresses increase from the ends to the middle of the truss, that the stresses in the main web members increase from the middle toward the ends, and that the counter members increase in stress toward the middle. The sizes of the various members are approximately proportional to their maximum stresses as above found, if snow and wind loads be disregarded.

It is also seen that the ratio of the minimum to the maximum stress is the same for all chord members, since the dead and live load stresses are proportional to the corresponding loads; the greatest chord stresses occurring when the bridge is fully loaded. Thus, if the dead load per foot be w and the live W, the minimum and the maximum chord stresses are, in general, in the ratio of w to $w + W$.

In the bracing the ratio for the range of stress is less than for the chords. For the Howe and Pratt systems the diagonals near the middle of the truss are at times unstrained, so that the ratio of minimum to maximum stress is zero. In the Warren truss the middle diagonals are sometimes in tension and sometimes in compression.

These facts are important in deciding upon the unit-stresses to be used in designing the members. As repeated stresses are the more dangerous the greater their range, the working unit-stress should be smaller for high ranges than for low ones. Accordingly higher unit-stresses may be used for the chords than for bracing, and higher unit-stresses may be used for the braces near the ends of the trusses than for those near the middle. For wrought iron in tension chord members are usually designed by taking 12 000 pounds per square inch as the working stress, while only about 8 000 or 9 000 pounds are allowed for the bracing.

In railroad bridges these considerations are more important

than for highway bridges, on account of the greater shock and more frequent application of the full live load.

Prob. 62. Find the maximum and mimimum stresses for all the members of a queen-post truss, the panel length being 10 feet, the depth 10 feet, the dead apex load on the lower chord 2 000 pounds, and the live apex load 8 000 pounds.

ART. 38. THE BOWSTRING TRUSS.

A truss with bent upper and horizontal lower chord, like Fig. 47, is called the 'bowstring,' and is a favorite form for highway bridges. It is built so that the verticals may take either tension or compression, while the diagonals take only tension. The principles of Art. 5 and Art. 7 suffice for the computation of its stresses, but the theorem $S = V \sec \theta$ for web members cannot be applied, since this is true only for trusses with horizontal chords.

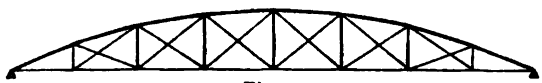

Fig. 47.

As an example, let the truss have 6 panels, each 15 feet long on the upper chord, the depths being, $Dd = 13$ feet, $Cc = 11.7$ feet, and $Bb = 7.5$ feet. The dead panel load, all on the lower chord, is 2.5 tons and the live panel load is 7.5 tons. Fig. 48 represents the truss with one set of diagonals removed.

To compute the maximum chord stresses the dead and live panel loads, 10 tons, are placed at each lower chord apex. Then for the member cd we have

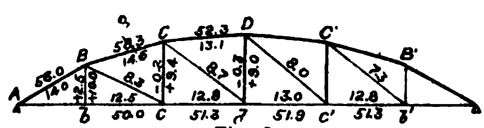

Fig. 48.

$$S \times 11.7 = 25 \times 30 - 10 \times 15,$$

whence $S = + 51.3$ tons. For CD we find the lever arm to be

12.92 feet, and the moment equation is

$$S \times 12.92 + 25 \times 45 - 10\,(30 + 15) = 0.$$

whence $S = -52.3$. The minimum chord stresses are one-fourth of the maximum stresses.

For the webbing the method of moments can also be used, the live load being placed for each member in the position to give the largest positive or negative shear (Art. 32). Thus, for the vertical *Dd* we cut it by a section, place the live load on the right, and take the center of moments at the intersection of *CD* and *cd*, which is 105 feet to the left of *A*. The reaction for this loading is

$$R = 6.25 + \frac{7.5}{6}(1 + 2) = 10 \text{ tons},$$

and the moment equation for *Dd* is

$$-10 \times 105 + 2.5\,(120 + 135 + 150) - S \times 150 = 0,$$

from which $S = -0.25$, which is the minimum stress. The greatest tension in *Dd* will occur when the truss is fully loaded and is 9 tons.

For the maximum stress in the diagonal *Cd* the live load is placed on the right, the center of moments is on the lower chord produced at 105 feet to the left of *A*, the lever arm of *Cd* is 92.27 feet, and the stress is found from the equation

$$-13.75 \times 105 + 2.5\,(120 + 135) + S \times 92.27 =$$

whence $S = +8.7$. For live load on the left the stress in *Cd* is 0 and the counter *Dc* comes into action. To find the stress for *Dc*, we have

$$-17.5 \times 105 + 10\,(120 + 135) - S \times 88.41 = 0,$$

from which $S = +8.0$ tons.

In the same manner all the other stresses are found and marked on the diagram. The method of resolution of forces can also be

used to find the stresses in the diagonals; the load being put on the truss in the proper position and the two adjacent chord stresses being found by moments, the difference of these is the horizontal component of the stress for the given diagonal. The bowstring truss is sometimes built without counter-ties, in which case the main ties take compression as well as tension, like the Warren truss.

Prob. 63. Compute the maximum and minimum stresses for the members *bc*, *Cc*, *Cb*, and *Bc* in Fig. 48.

ART. 39. THE PARABOLIC BOWSTRING TRUSS.

The apex points of the upper chord of a bowstring truss should be so arranged as to lie upon some regular curve, for evident æsthetic reasons. If this curve be a parabola the truss enjoys the remarkable property that under uniform load the diagonals are unstrained and the lower chord stresses are the same in all panels.

To prove this let d be the center depth and l the span. Then for a uniform load of w pounds per linear foot the lower chord stress at any distance x from the left support is

$$S = \frac{\frac{1}{2}wlx - \frac{1}{2}wx^2}{y}$$

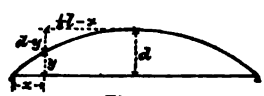

Fig. 49.

in which y is the lever arm for the lower chord at the section. To find the value of y consider that the equation of the parabola with reference to its vertex is

$$(\tfrac{1}{2}l - x)^2 = m(d - y),$$

and since $x = 0$ when $y = 0$, the parameter m equals $\frac{l^2}{4d}$. Hence,

$$y = \frac{4d}{l^2}(lx - x^2),$$

Inserting this in the expression for S, we find

$$S = \frac{wl^2}{8d}$$

and because this is constant the lower chord stresses are all the same. Now referring to Fig. 48, it is seen that the diagonals can have no stress under uniform load, for the horizontal component of the stress in any diagonal equals the difference of the chord stresses in adjacent panels.

If the span and center depth be given the above formula for y determines the depth at each panel point, so that the upper apex points may lie on a parabola. For instance, if $l = 90$ and $d = 13$, as in Fig. 48, we find $y = 7.22$ when $x = 15$ and $y = 11.55$ when $x = 30$. The upper chord apexes in that diagram lie upon some other curve than a parabola.

The diagonal stresses in a parabolic bowstring truss are therefore found by putting only the live load on the bridge in the proper position for each member. The maximum stresses in the verticals are found by adding one dead panel load to the largest live load stresses. If Fig. 48 be a parabolic bowstring truss of 90 feet span and 13 feet center depth, the maximum stresses due to the same loads as there used are: $Ab = bc = cd = 51.9$ tons, $Dd = Cc = Bb = 10$ tons, while the diagonals are strained only by live load.

The bowstring truss may also be used as a deck bridge, in which case the main stress in the verticals is compression. The diagram, Fig. 50, gives the maximum and minimum stresses for a parabolic deck bowstring truss of 80 feet span and 10 feet center

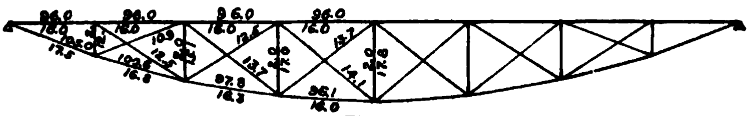

Fig. 50.

depth, the dead panel load being 2 tons and the live panel load 10 tons. It will be noticed that the maximum stresses in the

diagonals do not greatly vary, so that they can be made uniform in size, and that the verticals are always in compression.

Prob. 64. Compute the stresses for the deck parabolic bowstring truss in Fig. 50.

ART. 40. OTHER FORMS OF TRUSSES.

The lenticular truss shown in Fig. 51 is also used for highway bridges to some extent. The broken lines show the roadway and its connection to the trusses, the vertical end pieces being heavy posts, and the others tension rods. The roadway may also be placed higher and be attached directly to the verticals of the truss. The apexes of the chords may lie on parabolic or other graceful curves; if on parabolas the diagonals are unstrained under uniform load. The stresses are computed by the same methods as for the bowstring truss. The verticals are struts, and the diagonals ties as in the Pratt truss.

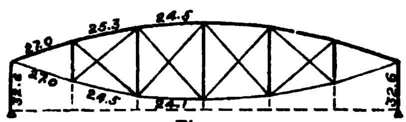

Fig. 51.

The bowstring truss may be also built with all its web members inclined, as in Fig. 52, which shows the through type. As this truss is used mainly for æsthetic reasons, it is not often built as a deck bridge. The diagonals must be designed to take both tension and compression.

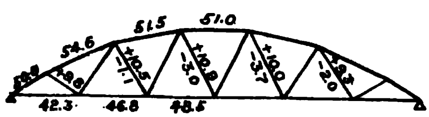

Fig. 52.

The maximum stresses shown on Figs. 51 and 52 are for a dead load of 0.27 tons and a live load of 0.816 tons per linear foot, the span in both cases being 60 feet and the apexes of the curved chords lying on circles whose radii is 50 feet. The center depth for the lenticular truss is 20 feet, and for the bowstring 9.75 feet;

(this depth of 20 feet is too great for good practice, and in fact Fig. 51 is drawn much less than this in depth).

The Pratt truss for spans greater than 175 to 200 feet is usually built with two systems of webbing, forming what is known as 'double intersection trusses,' the discussion of which is reserved for the next Chapter, where will also be described a number of other forms used for highway as well as for railroad bridges. The Warren truss is frequently built with a double system of webbing even for spans as short as 50 feet.

Prob. 65. A lenticular truss of 60 feet span, like Fig. 51, is 20 feet deep at the middle. Find the depths for the other verticals, taking the curves as circles of 50 feet radius. Find the maximum stresses due to a dead load of 0.27 and a live load of 0.816 pounds per linear foot per truss.

Ans. $Bb = -1.9$, $Cc = -3.2$, $Dd = -2.7$, $Bc = +5.6$,
$Cd = +6.9$, $Dc = +6.2$, $Cb = +4.7$

ART. 41. SNOW LOAD STRESSES.

Thus far the stresses have been regarded as caused only by the dead and live loads, and in fact many highway bridges have been built in which only these loads are considered. A complete investigation, however, must include the effects of snow and wind.

For highway bridges the snow load is taken from 0 to 20 pounds per square foot of floor surface, depending upon the climate where built. In cities the sidewalks are usually kept free from snow, and in the country the full live load is not apt to come upon the bridge when its floor is heavily loaded with snow. The value selected may hence generally be a little lower than for roofs.

As the snow is a uniform load the computation of the stresses caused by it is made in exactly the same manner as for dead load, all the weight being taken on that chord which supports the

floor. Or if w' and w be the snow and dead loads per linear foot, the snow load stresses can be found by multiplying the dead load stresses by $\frac{w'}{w}$, provided all the dead load be taken on the roadway.

Prob. 66. A through Pratt truss of 162 feet span has 9 panels, and is 24 feet deep; its width, including sidewalks, is 30 feet. Find the stresses caused by a snow load of 10 pounds per square foot.

ART. 42. WIND STRESSES.

The wind is to be taken as blowing horizontally at right angles to the line of the bridge, and exerting a pressure of from 35 to 40 pounds per square foot. For a highway bridge the surface exposed to wind action is usually taken as double the side elevation of one truss. If the area of this be not known an approximation to its value may be found by taking it as many square feet as there are linear feet in the skeleton outline of the truss.

The wind pressure takes effect in the lateral bracing, by which it is transferred to the abutments. Between the chords which support the floor, the floor beams act as struts and diagonal tension rods are attached to them near the trusses. Between the other chords normal struts and diagonal ties are generally used, making a lateral truss of the Pratt type. The chords of the main trusses are hence also the chords for the lateral bracing. Thus, Fig. 53 shows the plan of a bridge, *ab* and *cd* being the two chords. When the wind blows in the direction of the full arrows it produces tension in *ab* and compression in *cd*, and strains the full line diagonals. When it blows in the opposite direction it produces tension in *cd*, compression in *ab*, and strains the other set of diagonals.

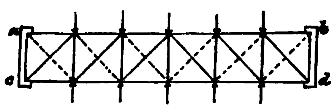

Fig. 53.

The wind is to be regarded as a live load; hence, for the chords it is to be applied at all the panel points, and for the bracing so placed as to give the greatest stress in each member.

For example, take a through Pratt truss highway bridge of 162 feet span and 18 feet width. Let $AabB$ be one-half of its side elevation, each panel being 18 feet long and 24 feet deep. Let $aa'b'b$ be the plan of the upper and $AA'B'B$ the plan of the lower lateral bracing. $Aaa'A$ is an end view showing the portal bracing which helps to transfer the wind pressure on the upper chord down to the abutments. It is required to compute the stresses due to a wind pressure of 38 pounds per square foot.

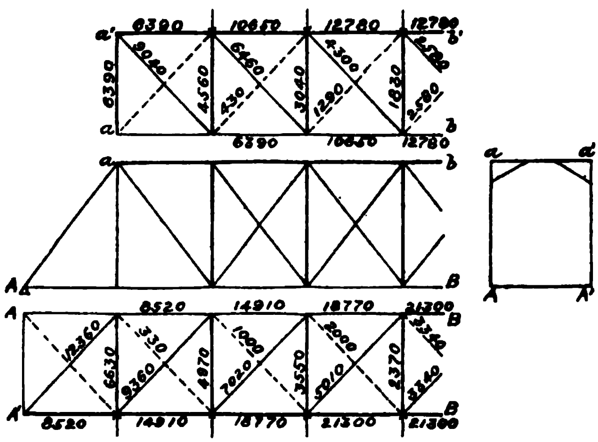

Fig. 54.

By the approximate rule the number of square feet exposed to wind action is,

$$162 + 126 + 8 \times 24 + 14 \times 30 = 900 \text{ square feet.}$$

The total wind pressure is then $900 \times 38 = 34\,200$ pounds, which gives about 2 130 pounds for each panel wind load.

Taking the wind as blowing in the direction of the arrows, shown in Fig. 54, and as uniformly distributed, the chord stresses are found as for dead load (Art. 27), the broken diagonals being unstrained.

Regarding the wind as a live load the largest shear for each

member of the bracing is found by Art. 32, and this multiplied by the secant (1.4142 for the ties and 1 for the struts) gives the greatest stress. When the wind travels from right to the left the full line ties are strained, and when in the other direction the broken ones.

Thus, all the wind stresses are found and marked on the diagrams. When the wind blows on the other side of the bridge, the diagonal and chord stresses are to be interchanged, while the strut stresses remain the same. As the diagonal lateral ties are made adjustable to stiffen the trusses together, they are often larger in section than the wind stresses require in order to allow for initial tension.

Occasionally the lateral bracing is made of the Howe type, the diagonal pieces being struts, and the normal ones ties, but for iron bridges the usual style is the Pratt, as above explained. In deck bridges, ties called 'sway-bracing' are often introduced, which run from each upper chord to the opposite lower chord; these serve to stiffen the bridge under the live load, and also at the ends perform the same function as the portal bracing.

As the effect of wind is to produce tension in one chord and compression in the other, the dead and live load stresses will be both increased and diminished, so that the range of stress is larger. It is, however, not probable that a highway bridge would be covered with the live load during a hurricane which causes a pressure of 40 pounds per square foot.

Prob. 67. Find the stresses due to wind in both lateral systems of a through Pratt truss, the span being 150 feet, panel length 15 feet, depth 20 feet, width between trusses 16 feet.

ART. 43. FINAL MAXIMUM AND MINIMUM STRESSES.

The final maximum stress in a truss member is the largest resulting from all possible combinations of dead, live, snow and wind loads, and the minimum stress is the smallest that can occur from the action of these loads. The maximum and minimum stresses found in Arts. 31–39 are for dead and live loads only. With these are now to be combined the snow and wind load stresses.

To illustrate, we take the through Pratt truss already computed for dead load in Art. 28 and for wind load in Art. 32. The span is 162 feet, panel length 18 feet; depth 24 feet, width 18 feet, dead apex load on upper chord 1.35 tons and on lower chord 3.3 tons, live apex load 6.4 tons, snow apex load 0.8 tons, and wind apex load 1.065 tons. The stresses due to dead, live, snow, and wind loads having been computed, they are tabulated as follows, and the final maximum and minimum stresses found by addition. First, for the lower chord, we have,

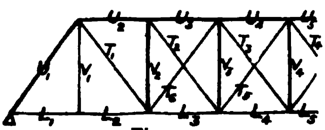

Fig. 55.

	L_1	L_2	L_3	L_4	L_5
From dead load	+ 14.0	+ 14.0	+ 24.4	+ 31.4	+ 34.9
From live load	+ 19.2	+ 19.2	+ 33.6	+ 43.2	+ 48.0
From snow load	+ 2.4	+ 2.4	+ 4.2	+ 5.4	+ 6.0
From east wind	— 4.3	— 7.5	— 9.4	— 10.2	— 10.2
From west wind	0.0	+ 4.3	+ 7.5	+ 9.4	+ 10.2
Maximum stress	+ 35.6	+ 39.9	+ 69.7	+ 89.4	+ 99.1
Minimum stress	+ 9.7	+ 6.4	+ 15.0	+ 21.2	+ 24.7

For the end post and upper chord we have in like manner

	U_1	U_2	U_3	U_4	U_5
From dead load	— 23.3	— 24.4	— 31.4	— 34.9	— 34.9
From live load	— 32.0	— 33.6	— 43.2	— 48.0	— 48.0
From snow load	— 4.0	— 4.2	— 5.4	— 6.0	— 6.0
From east wind		— 3.2	— 5.3	— 6.4	— 6.4
From west wind		0.0	+ 3.2	+ 5.3	+ 6.4
Maximum stress	— 59.3	— 65.4	— 85.3	— 95.3	— 95.3
Minimum stress	— 23.3	— 24.4	— 28.2	— 29.6	— 28.5

For the web members there are no wind load stresses, and

	T_1	T_2	T_3	T_4	T_5	T_6
From dead load	+ 17.4	+ 11.6	+ 5.8	0.0	0.0	0.0
From live load	+ 24.9	+ 18.6	+ 13.3	+ 8.9	+ 5.3	+ 2.7
From snow load	+ 3.0	+ 2.0	+ 1.0	0.0	0.0	0.0
Maximum stress	+ 45.3	+ 32.2	+ 20.1	+ 8.9	0.0	0.0
Minimum stress	+ 16.5	+ 8.9	+ 0.5	0.0	0.0	0.0

The counters T_5 and T_6 have no stress, but the former at least should be inserted to stiffen the panel and assist in emergencies. It will be noticed that the minimum stress in any main tie is less than that due to the dead load by the amount of compression caused by the live load; thus, for T_2 the minimum tension is $11.6 - 2.7 = 8.9$ tons.

In conclusion it should be said that it is not the common practice to use the wind stresses for finding the maximum and mimimum chord stresses, it being generally assumed that the live load will not come upon the bridge in violent storms. The many failures of highway bridges indicate, however, that they have not been built of sufficient strength, and hence it is certainly to be recommended that the true maximum and minimum stresses should in all cases be computed.

Prob. 68. Find the final maximum and minimum stresses for the verticals, in the above example.

Prob. 69. A through Warren truss of 60 feet span has 10 panels; its depth is 12 feet, width between trusses 16 feet, and it has two sidewalks each 5 feet wide. The dead load per linear foot is to be found from formula (2) and all to be taken on the lower chord. The live load is 90 and the snow load 10 pounds per square foot of floor. The wind pressure is to be 40 pounds per square foot. Compute the final maximum and minimum stresses.

ART. 44. INVESTIGATION AND DESIGN.

The remarks made in Arts. 21 and 22 concerning the investigation and design of roofs apply, with but slight modifications, to highway bridges. Each of the problems is a complex one, requiring great skill in many other departments besides the mere computation of the stresses. The designing of the joints, in particular, is often a difficult task in order to secure compactness, stability, and economy, and in large establishments is entrusted only to experts. The student should form the habit of sketching the details of bridge connections in order to become familiar with the various designs in use.

The flexural stresses due to the weights of the members themselves are too small to be considered, but when floor beams are placed on the chords, as in the Howe truss, the effect of flexure must be computed by the methods of Art. 74 or 75, Mechanics of Materials. The chord, if continuous, may be regarded as a beam fixed at the panel points.

The end supports of bridges are arranged like those of roofs, one end of each truss being fastened to the abutment, while the other is free to move longitudinally to allow of expansion and contraction under changes of temperature.

Prob. 70. What will be the change in length of an iron bridge of 200 feet span between + 90 and — 20 degrees Fahrenheit?

CHAPTER III.

RAILROAD BRIDGE TRUSSES.

ART. 45. DEAD LOADS.

The bridges here discussed will be those for a standard gauge railroad, single or double track. The clear width between the two trusses of a through bridge is for a single track about 13 or 14 feet, and for a double track about 25 feet. Only a few two-truss bridges for three tracks have been built. Bridges with four tracks have three or more trusses.

The dead load, or own weight, of such a bridge depends upon its span, the live loads carried, the unit-stresses adopted, the style of the bridge and many other considerations, so that it can only be accurately ascertained for a particular case by actual computation. The following values give, however, approximate weights which may be used as a guide in the absence of more detailed information.

The floor system consists of rails, guard rails and cross-ties, which rest upon stringers, these in turn being supported by the floor beams, which are attached to the chords at the panel points. The weight of the rails, guard rails and cross-ties averages about 300 pounds per linear foot of track, but in computing stresses it is often taken as high as 400 pounds. The stringers and floor beams vary in weight with the panel length. The weight of the entire floor system for a single track bridge may be stated at from 450 to 600 pounds per linear foot.

The lateral bracing of both upper and lower chords will weigh from 50 to 300 pounds per linear foot of single track, depending upon the length of span.

The total dead load of the bridge can be roughly found from the following empirical formulas, which for spans less than 300 feet will give values sufficiently accurate for the computation of stresses.

$$\begin{array}{ll} \text{For single track,} & w = 560 + 5.6l, \\ \text{For double track,} & w = 1\,070 + 10.7l, \end{array} \qquad (3)$$

in which l is the span in feet, and w is the dead load in pounds per linear foot. Wooden bridges weigh about the same as iron ones of equal strength.

Prob. 71. If the floor system weighs 600 and the lateral bracing 150 pounds per linear foot, find the approximate weight of the trusses for a span of 250 feet.

Prob. 72. A distance of 720 feet between two abutments is to be spanned with single track bridges. If each pier cost $8,000, and the bridges cost 4 cents per pound, find the most economical number of spans.

ART. 46. LIVE LOADS.

The live load to be assumed in computing the bridge is that of the heaviest cars and locomotives which pass, or are to pass over it. It is often a very difficult matter to ascertain these data, but when a bridge is to be built the railroad company usually specifies the live loads to be used, so that the designer is free from that responsibility. Several methods of stating the live load are in use:

1st—A uniform live load varying from 2 200 to 4 500 pounds per linear foot of track, the largest load being for the shortest span, and about as follows:

Span,	50	100	150	200	300	400	500 feet,
Load,	4 200	3 600	3 200	3 000	2 600	2 400	2 200 pounds.

This method was formerly much in use, but is now only occasionally employed for computing the chord stresses.

2d—A uniform train load, varying as above, which is preceded by one panel of heavy locomotive load. The preceding heavy load is taken about 65 000 pounds for a ten foot panel, and is increased 3 500 pounds for each foot of increase in panel length; or if p be the length of the panel this load is for a single track, $30\,000 + 3\,500\ p$. Sometimes the preceding locomotive load is taken for two or three panels in front of the train instead of for one. Some railroad companies use the uniform live load only for computing the chords, while for the webbing the preceding load is specified.

A number of other varieties of loading, due to single concentrated weights and locomotive wheels, are also in use. These will be explained in Arts. 57–59.

If the bridge has two tracks each truss sustains the loads as given, for it might happen that both tracks would be covered at the same time. If the bridge has but one track the stated loads are to be divided by two to obtain the loads per truss.

The reason why the largest loads are taken for the shortest spans is evident. A bridge of 50 feet will be entirely covered by a locomotive and tender, while one of long span would rarely be loaded heavier than by a train drawn by two locomotives.

Prob. 73. Find the train and locomotive panel loads for the truss of a single track bridge of 252 feet span with 18 panels.

ART. 47. SNOW, WIND AND IMPACT.

The snow load is not considered for railroad bridges, since the floor is open so that little can be retained.

The wind is to be taken at from 30 to 40 pounds per square foot on double the side elevation of one truss, to which should be added the side surface of a train standing upon the track. This train surface is about 10 square feet for each linear foot of the bridge. All the stress computations for wind are made exactly as in Art. 42, the lateral bracing of those chords which supports the floor taking all the wind load upon the train. The lateral bracing is almost always of the Pratt type, and rarely has a double system of diagonals.

Impact is the effect of suddenly applied loads which, as is well known, produce greater stresses than the same static loads. It is also generally taken to include the effect of shocks due to irregularities in the track, to unbalanced driving wheels and other causes. The manner in which the impact stresses are to be determined and allowed-for is an unsettled problem. Evidently, however, the effect is greater the shorter the span, since the ratio of live to dead load is higher for shorter spans than for long ones, and for the same reason it is greater in the webbing than in chords (Art. 37).

Some engineers allow for impact by increasing the given static loads by a percentage which decreases with the span, being about 20 per cent. for 50 feet spans, 10 per cent. for 100 feet spans, and becomes 0 for 150 feet spans. Others compute the maximum stresses due to given loads and then increase them by similar varying percentages, taking less increase for chords than for webbing. Others again use the maximum computed stresses and allow for the impact by varying the working unit-stresses, taking the lowest unit-stresses for the members having the greatest range of stress.

The last method will be adopted in this book, as it seems on the whole to be the most rational. In this Chapter we have then to compute the greatest and least stresses in the truss members due to the assigned dead, live and wind loads.

Prob. 74. Find the approximate ratio of live to dead load for a bridge of 100 feet span, and also for a bridge of 300 feet span.

ART. 48. KINDS OF TRUSSES.

All of the statical principles in Chapter I, and all of the general methods of Chapter II, are directly applicable to the computation of stresses in railroad bridge trusses. The Howe, Pratt and Warren trusses are extensively used on railroads as well as on highways. For short spans the king and queen-post trusses are now seldom used, their place being generally filled by wrought iron riveted plate girders.

The wooden Howe truss, once a favorite form, on account of simplicity of construction, lacks in stiffness and durability, and is hence going gradually out of use. The Howe, built wholly in iron, is called the Jones; very few of these have been constructed as it is not economical to have numerous iron struts in an inclined position, and there is difficulty in making good end connections. The ill-fated Ashtabula bridge was a Jones truss.

The Pratt was originally made all of wood, except the wrought iron diagonals and cast-iron joint connections. Now it is built entirely of wrought iron and is more extensively used than any other kind. A Pratt form with a double system of webbing, as shown in Fig. 60, is called the Whipple truss. This duplication of the web system is made in long spans for the purpose of keeping the panel points and floor beams nearer together than would be the case in a single system with the same inclination of diagonals.

The through Warren truss is sometimes built with vertical suspension rods, each of which supports a floor beam, and takes only the stress from the panel load, which is thus virtually supported by the upper chord. If inverted this becomes a deck truss in which the vertical members are compression pieces. As these verticals are not real truss members, they may be called 'sub-verticals.'

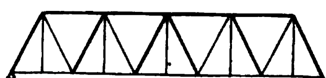

Fig. 56.

The Warren truss is frequently built with a double and occasionally with a quadruple system of webbing. If the joints of these are riveted, as is generally the case for short spans, they are called lattice girders. A lattice may be regarded as a modification of the plate girder, with portions of the web omitted.

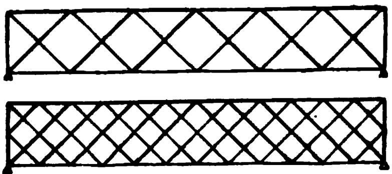

Fig. 57.

A number of other forms of trusses will be explained in the following Articles of this Chapter. Truss bridges of less span than 50 feet are now rarely built for railroad service. For spans of from 5 to 20 feet solid I beams are used, and for spans of 20 to 50 feet plate girders. For spans of 50 to 100 feet riveted lattice girders are a favorite type, and for spans above 100 feet pin-connected trusses are most common in this country.

Prob. 75. Find the stresses in Fig. 56, due to a load of 2 tons at each panel point of the lower chord.

ART. 49. THE WARREN TRUSS WITH SUB-VERTICALS.

The Howe, Pratt and Warren trusses for railroad bridges, when with a single system of webbing, are computed, if the live load be uniform, in exactly the same manner as set forth in the last Chapter. The following example illustrates the method when the uniform live load is preceded by a heavy locomotive panel load.

Let the truss be a Warren with vertical struts, 100 feet in span and 10 feet in depth. Let it be a deck single track bridge weighing 1 120 pounds per linear foot; let the live load be a train of 3 600 pounds per linear foot, preceded by two panels of locomotive load weighing 64 000 pounds each. It is required to compute the maximum and minimum stresses due to these loads.

For brevity we find the panel loads in short tons; the dead panel load is 2.8 tons, the train panel load is 9.0 tons and the locomotive panel load 16.0 tons.

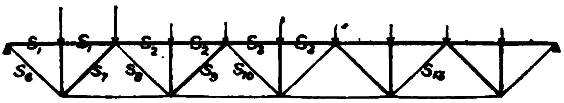

Fig. 58.

For the verticals the minimum stress is evidently the dead panel load 2.8 tons, and the maximum is the full panel load 2.8 + 16.0, or 18.8 tons, both compression.

The minimum chord stresses are due to dead load alone; placing the load 2.8 tons at each panel point of the upper chord the stresses are computed by either of the methods of Art. 27. The maximum chord stresses are due to dead plus live load; or to seven loads of 11.8 tons and two loads of 18.8 tons placed as shown in Fig, 58; for these loads the stresses are found by the method of moments.

The maximum stress in any diagonal occurs when the dead load covers the whole bridge and the live load is placed on the right of a section cutting that diagonal. The shear for these loads being found by Art. 33, this shear multiplied by the secant of the angle which the diagonal makes with the vertical gives the stress (Art. 26). For the minimum stress the load is reversed in direction and covers the truss on the left of the section; or if preferred the train may be backed and the maximum shear found for the corresponding member in the right hand part of the truss; thus the maximum shear for S_{13} has the same numerical value as the minimum shear for S_8.

The following equations for finding the stresses in a few of the pieces will serve as examples of the methods:

For min S_3, $S \times 10 = 4.5 \times 2.8 \times 50 - 2.8\,(40 + 30 + 20 + 10)$

For max S_3, $S \times 10 = 65 \times 50 - 18.8(40 + 30) - 11.8(20 + 10)$,

For min S_8, $S = -[12.6 + 16(\frac{1}{10} + \frac{2}{10}) - 2.8 \times 7] \times 1.4142$,

For max S_8, $S = [12.6 + 9(\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} + \frac{5}{10}) + 16(\frac{6}{10} + \frac{7}{10}) - 2.8 \times 2] \times 1.4142$.

Thus all the required stresses are found; for the chords,

	S_1	S_2	S_3	S_4	S_5
Maximum stress	− 65.0	− 138.6	− 158.0	+ 111.2	+ 154.2
Minimum stress	− 12.6	− 29.4	− 35.0	+ 22.4	+ 33.6

and for the diagonals,

	S_6	S_7	S_8	S_9	S_{10}
Maximum stress	+ 91.9	− 74.5	+ 58.4	− 43.5	+ 30.0
Minimum stress	+ 17.8	− 11.6	+ 3.1	+ 6.6	− 17.8

Here, as usual, plus denotes tension, and minus, compression.

Prob. 76. Compute the maximum and minimum stresses for a Howe truss of 100 feet span, 8 panels and 20 feet depth, using the same dead and train loads per linear foot as in the above example, the preceding locomotive panel load being 75 500 pounds over one panel only.

ART. 50. THE DOUBLE SYSTEM WARREN TRUSS.

In a truss with two diagonal systems a section in general will cut four pieces, and as there are only three conditions of equilibrium (Art. 4), it would at first appear that the stresses could not be determined. This difficulty is overcome by the following fourth condition or hypothesis:

Each system of diagonals is strained only by the loads which rest upon it.

Thus, in Fig. 59, the loads P are transferred to the abutments by the diagonals drawn full, while the loads Q are carried by the broken diagonals. Hence, such a truss consists of two independent systems or trusses,

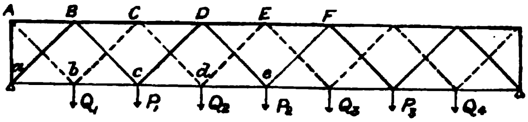

Fig. 59.

the chords being common to both, the stresses for which may be separately found. The stress in any chord member as BC is then the sum of the separate stresses in AC and BD.

For example, let the truss in Fig. 59, be 100 feet in span, 12.5 feet in depth, and all the loads be on the lower chord. Let the dead panel load per truss be 3.5 tons, and the train panel load 12 tons, preceded by one locomotive panel load of 19 tons. To find the minimum stresses in the upper chord each of the loads P and Q is 3.5 tons. For the full line system we consider only the loads P_1, P_2 and P_3; the reaction is 5.25 tons, and the equation for stress in DF is,

$$-DF \times 12.5 = 5.25 \times 50 - 3.5 \times 25, \quad DF = -14.0,$$

and in the same manner we find $BD = -10.5$, $AB = 0$. Again, for the broken line system we consider only the loads Q_1, Q_2, Q_3 and Q_4; the reaction is 7 tons, and for stress in CE,

$$-CE \times 12.5 = 7 \times 37.5 - 3.5 \times 25, \quad CE = -14.0,$$

and in the same way we have $AC = -7.0$. Then for the final minimum stresses we find,

$$AB = -0 - 7.0 = -7.0, \qquad BC = -7.0 - 10.5 = -17.5,$$
$$CD = -10.5 - 14.0 = -24.5, \quad DE = -14.0 - 14.0 = -28.0.$$

In the same way the minimum stresses in the lower chord are found to be,

$$ab = +5.25, \; bc = +15.75, \; cd = +22.75, \; de = +26.25.$$

For the maximum chord stresses the locomotive should stand at Q_1 and the other panel points receive the train load; thus $Q_1 = 22.5$ tons, and the other loads 15.5 tons. For the full system the reaction is 23.25 tons and for the broken system 37.125 tons. The stresses for each system are now found as before and added, giving,

$AB = -37.1$, $BC = -83.6$, $CD = -112.9$, $DE = -128.4$,
$ab = +23.25$, $bc = +75.0$, $cd = +106.0$, $de = +119.75$.

The ratios of minimum to maximum stress are here only approximately in the ratio of dead to total load, on account of the single heavy panel weight.

The maximum and minimum diagonal stresses are found by the usual method, each system carrying only its own loads. For the maximum in Cd, we have,

$$S = [7 + 19 \times \tfrac{6}{8} + 12\ (\tfrac{5}{8} + \tfrac{1}{8}) - 3.5] \times 1.4142 = 30.2 \text{ tons},$$

and for the minimum,

$$S = [7 + 19 \times \tfrac{7}{8} - 3.5 - 19] \times 1.4142 = 1.6 \text{ tons}.$$

Thus the following stresses are determined

	Ab	Bc	Cd	De	Ba	Cb	Dc	Ed
Max.	+52.5	+40.3	+30.2	+20.2	−40.3	−30.2	−20.2	−12.2
Min.	+9.9	+7.4	+1.6	−4.2	−7.4	−1.6	+ 4.2	+12.2

The minimum compressive stress in the end post Aa is 7 tons and the maximum is 37.1 tons.

Prob. 77. A double system deck Warren truss of 100 feet span has 10 panels and is 10 feet deep. The dead load per linear foot per truss is 560 pounds, and the train load 1 800 pounds, which is preceded by two heavy locomotive panel loads of 65 000 pounds each. Compute the maximum and minimum stresses in all members.

ART. 51. THE WHIPPLE TRUSS.

The Whipple truss, or double intersection Pratt, is very extensively used. The example is a through double track bridge of 127 feet $10\frac{1}{2}$ inches span, and having 11 panels, each 11 feet $7\frac{1}{2}$ inches long, and 23 feet 3 inches deep. The dead load per linear foot per truss is 1 000 pounds, the train load 3 000 pounds, and the locomotive load 4 500 pounds. The locomotive load is not to be used for the chords, but for the webbing one panel of it precedes the train. These data give the dead panel load 11 625 pounds, the train panel load 34 875 pounds, and the locomotive panel load 52 312 pounds. Fig. 60 shows the truss with the counter-ties in broken lines.

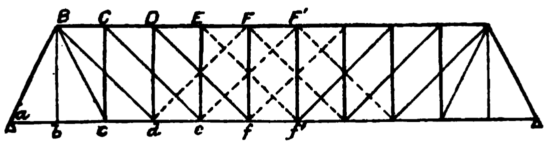

Fig. 60.

As this truss has an odd number of panels, the division into two separate systems cannot be made so that both will be symmetrical on each side of the middle. Under uniform load, however, it is evident that there is no shear in the middle panel *Fff'F'*; hence, the shear in *Df* is one panel load, as also is *Ce*. Therefore, to find the chord stresses the division into systems must be made as shown in Fig. 61. By the method of moments the chord stresses are now found as in the last Article. Thus, for *de* we have to find *df* from the first system and *ce* from the second. For the first the reaction under full load is 139 500 pounds and for the second 93 000 pounds. Then,

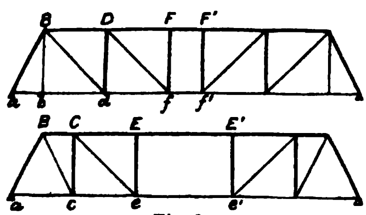

Fig. 61.

$$df \times 23.25 = 139\,500 \times 1\tfrac{1}{2} \times 23.25 - 46\,500 \times 1 \times 23.25,$$
$$ce \times 23.25 = 93\,000 \times 23.25,$$

whence $df = +\ 162\ 750$ and $ce = +\ 93\ 000$, the sum of which is 255 750, the maximum stress in *de*.

The same result may be found by the method of increments which is perhaps shorter for this case. In Fig. 60, we see that the stress in *de* is the sum of the horizontal components of the stresses in *Ba*, *Bc* and *Bd*. The shears for these members are 232 500, 93 000 and 93 000 pounds; the tangent for *Ba* and *Bc* is $\frac{11.625}{23.27} = 0.5$, and for *Bd* is 1.0. Hence, for the maximum stress in *de* we have

$$de = (232\,500 + 93\,000) \times 0.5 + 93\,000 \times 1.0 = +255\,750.$$

Thus are found the final stresses as follows:

	ab and *bc*	*cd*	*de*	*ef*	*ff'*
Maximum	+ 116 250	+ 162 750	+ 255 750	+ 302 250	+ 348 750
Minimum	+ 29 060	+ 40 690	+ 63 940	+ 75 560	+ 86 690

For the upper chord *BC* has the same stress as *de*, *CD* the same as *ef*, and *DF'* the same as *ff'*.

For the diagonals the maximum and minimum stresses occur under unsymmetrical loads, and hence some ambiguity exists as to the manner in which the division into systems is to be made. For the dead load the division shown in Fig. 61 is certainly correct, but an unsymmetrical live

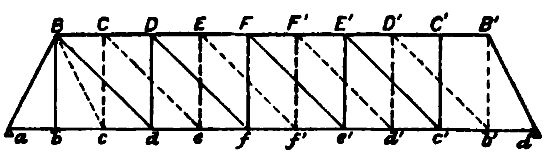

Fig. 62.

load must be regarded as transferred in a different manner. Let Fig. 62 be drawn showing diagonals inclined in one direction only, let the live load come on from the right, and suppose each system to carry only the live loads which rest upon it. For maximum stress in *Dd* the locomotive is at *f* and the train at *e'* and *c'*; then

$$-Dd = 11\,625 + 52\,312 \times \tfrac{6}{11} + 34\,875\,(\tfrac{4}{11} + \tfrac{2}{11}$$

whence max $Dd = -59\,180$. This multiplied by sec θ gives the stress in Df.

For the counter Fd or $F'd'$ the minimum stress is 0, and the maximum is,

$$F'd' = (-11\,615 + 52\,312 \times \tfrac{3}{11} + 34\,875 \times \tfrac{1}{11}) \times 1.4142.$$

For Bd the minimum stress is,

$$Bd = (2 \times 11\,615 - 52\,312 \times \tfrac{1}{11})\ 1.4142,$$

and the maximum for Bc is,

$$Bc = [2 \times 11\,625 + 52\,312 \times \tfrac{9}{11} + 34\,875 \cdot (\tfrac{7}{11} + \tfrac{5}{11} + \tfrac{3}{11} + \tfrac{1}{11}] \times 1.118.$$

Thus we find all the web stresses, using Fig. 61 for the dead load and Fig. 62 for the moving live load. The stresses in Ba and Bb are found in the same manner as if the truss were a single system. The following are the final values: for the end post and verticals,

	Ba	*Bb*	*Cc*	*Dd*	*Ee*	*Ff*
Maximum	−277 620	+63 940	−73 440	−59 180	−36 460	−25 360
Minimum	−64 980	+11 625	−11 625	−11 625	0	0

and for the diagonals,

	Bc	*Bd*	*Ce*	*Df*	*Ef'*	*F'e*	*Fd*
Max.	+130 540	+140 490	+103 850	+83680	+51550	+35860	+8 220
Min.	+27 400	+26 150	+2 990	0	0	0	0

The above chord stresses may be both increased and diminished by the wind pressure, which is here not considered.

Prob. 78. Compute the maximum and minimum stresses for a through Whipple truss of 10 panels, each 12 feet long and 24 feet deep, the dead load per linear foot per truss being 1 000 pounds, the train load 3 000 pounds, and for the webbing one locomotive panel load of 64 000 pounds to precede the train.

ART. 52. THE BOLLMAN TRUSS.

This form of truss, formerly built to some extent for short spans in the Western States, is shown in Fig. 63. It consists of a series of inverted unsymmetrical king-post trusses, *aBl*, *aCl*, etc., each of which carries the load resting upon it. Counter-ties shown by broken lines are placed in each panel to stiffen the structure. The upper chord and verticals are compression members and all the others are ties.

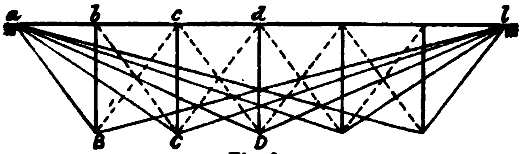

Fig. 63.

A load placed at any panel point, as *c*, is directly carried to the abutments by the ties *Ca* and *Cl*, and does not affect the other members. The stress in each post is evidently due to the panel load upon it. The maximum stress in the upper chord will occur under the full train load when all the main ties are brought into action. The maximum stress in any main tie occurs when the locomotive stands upon the corresponding post. The counters have no static stresses which can be computed, provided the main ties act in the manner supposed.

Let the span be 90 feet, the panel length 15 feet, the number of panels 6, the depth 20 feet, the dead panel load 6 tons, the train panel load 14 tons, and the locomotive panel load 22 tons.

The compressive stress in each vertical is 6 tons minimum and 28 tons maximum. The stress in any tie, as *Ca*, is the shear from the load on the vertical *Cc* multiplied by the secant of the angle which it makes makes with that vertical, thus,

$$\min\ Ca = \tfrac{4}{6} \times 6 \times \frac{\sqrt{20^2 + 30^2}}{20} = +\ 7.2 \text{ tons},$$

$$\max\ Ca = \tfrac{4}{6} \times 28 \times 1.803 = +\ 33.7 \text{ tons}.$$

The minimum chord stress is found by placing the dead load at each panel point. By moments we take the center of moments

for each load at the foot of its vertical, and have

$$-S \times 20 = \tfrac{5}{6} \times 6 \times 15 + \tfrac{4}{6} \times 6 \times 30 + \tfrac{3}{6} \times 6 \times 45 + \tfrac{2}{6} \times 6 \times 60 + \tfrac{1}{6} \times 6 \times 75.$$

For the maximum chord stress 28 tons are placed at *b* and 20 tons at each of the other panel points.

Thus the following values are found:

	al	*Bb*	*Ba*	*Ca*	*Da*	*Cl*	*Bl*
Maximum	− 92,5	− 6.0	+ 29.2	+ 33.7	+ 34.5	+ 29.5	+ 18,1
Minimum	− 26.2	− 28.0	+ 6.2	+ 7.2	+ 7.4	+ 6.3	+ 3.9

When the Bollman truss is used as a through bridge the roadway is suspended from the lower apexes *B*, *C*, *D*. The form is now rarely built as there are many theoretical objections against it.

Prob. 79. Prove that the stress in the upper chord of a Bollman truss due to dead load is $\frac{(n^2 - 1)\,p\,W}{6d}$, where n = number of panels, p = panel length, d = depth and W = panel load.

ART. 53. THE FINK TRUSS.

This truss, like the Bollman, has no lower chord, and consists of a series of inverted king-post trusses arranged one within the other as seen in Fig. 64, the primary being *aEl*, the secondary

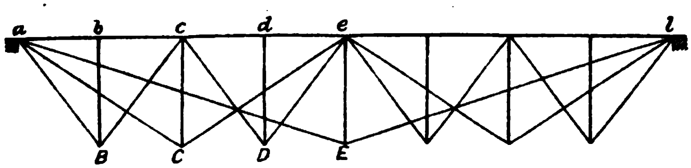

Fig. 64.

aCe, and tertiary *aBc*, *cDe*, etc. Diagonal counters *Cb*, *Cd*, etc., may also be placed in the panels to stiffen the structure. The vertical members are struts and all the diagonals are ties. From the method of construction it is plain that the ties perform the functions of a lower chord, and hence that the maximum stresses in all members occur when the truss is fully loaded.

Let the span in Fig. 64 be 80 feet, the panel length 10 feet and the depth 15 feet. Let the dead load per panel be 1.5 tons and the live load 9 tons, both on the upper chord. The minimum stresses will be $\frac{1.5}{10.5} = \frac{1}{7}$th of the maximum stresses so that it is only necessary to compute the latter.

Placing the total load 10.5 tons at each panel point, the stress in *Bd* and *Dd* is 10.5 tons compression. The stresses in *Ba*, *Bc*, *Dc*, *De* are each equal to $\frac{1}{2} \times 10.5 \times \sec\theta$, or to $+\ 6.3$ tons, since $\sec\theta$ is 1.202. The stress in *Cc* is 10.5 tons from the load upon it, plus one-half of 10.5 tons brought by the tie *Bc*, plus the same amount brought by *Dc*, or in total $-\ 21$ tons. This is divided between *Ca* and *Ce*, the stress in each of which is 10.5×1.667 or $+\ 17.5$ tons. In like manner the stress in *Ee* is $-\ 42$ tons and in *Ea* $+\ 59.8$ tons.

For the stress in any panel of the chord *ae* either the method of increments or the method of moments may be used. By the latter we have

$$S = \frac{1}{2} \times 10.5 \times \frac{10}{15} + \frac{1}{2} \times 21 \times \frac{20}{15} + \frac{1}{2} \times 42 \times \frac{40}{15} = 73.5.$$

This completes the determination of the maximum stresses, except for counter-ties *Ed*, *Cd*, etc., which cannot act under normal conditions on account of the absence of a lower chord. It is well, however, to insert such counters to provide against accidents. This form of truss is going out of use.

Prob. 80. Find the stresses for Fig. 64, if the span be 100 feet the depth 20 feet, the dead load per panel 2 tons, and the train load 14 tons, preceded by one panel of locomotive load weighing 22 tons.

Prob. 81. If a load be at *b* in Fig. 64, what part of it goes to each abutment, and how?

ART. 54. THE BALTIMORE TRUSS.

This modification of the Pratt type was introduced in order to avoid long panel lengths. In the deck form given in the figure all the verticals are struts, and all the diagonals ties, the counters being shown in broken lines. The members Bb, Dd, etc., are sub-verticals that are strained only by the panel loads which they directly bear.

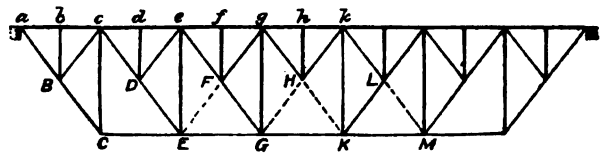

Fig. 65.

Let the span be 140 feet, having each panel of the upper chord 10 feet in length, and the depth 20 feet. The dead panel load is 8.4 tons, the live 8 tons, and the locomotive 15 tons, which is to be taken for two panels preceding the train. It is required to find the maximum stresses in all members.

The posts Bb, Dd, Ff and Hh have each a maximum stress of $3.4 + 15 = 18.4$ tons compression.

The maximum chord stresses occur when the points b and c have the locomotive load, and the other points the train load. The left reaction for dead and live loads then is,

$$R = 6\tfrac{1}{2}\,(3.4 + 8) + (\tfrac{13}{14} + \tfrac{12}{14})\,(15 - 8) = 86.6 \text{ tons.}$$

For the stress in CE the center of moments is at c, and

$$S \times 20 = 86.6 \times 20 - 18.4 \times 10, \quad \text{whence } S = +77.4.$$

For the stress in cd the center of moments is at E, and

$$-S \times 20 = 86.6 \times 40 - 18.4\,(30 + 20), \qquad S = -127.2,$$

which is the same as the stress for de. Thus are found the following maximum chord stresses,

$$ac = -86.6, \qquad ce = -127.2, \qquad eg = -148.5,$$
$$CE = +77.4, \qquad EG = +121.5, \qquad GK = +142.8.$$

For the upper part of any main tie, as Dc, the maximum stress occurs when the locomotive covers d and e and the train is at all points on the right; with this position the shear in Dc due to dead and live loads is,

$$V = 4\tfrac{1}{2} \times 3.4 + \frac{15}{14}(11 + 10) + \frac{8}{14}(9 + 8 + \ldots + 1) = +63.51,$$

hence the stress $Dc = 63.51 \times 1.414 = +89.8$ tons. In the same manner we find $Ba = +122.5$, $Fe = +60.4$ and $Hg = +34.2$.

For the lower part of any main tie, as DE, the locomotive stands at e and f, with the train on the right; here, however, the method of shears apparently fails, as a vertical section through DE cuts four pieces, and for this section,

$$DE \cos\theta = De \cos\theta + V, \quad \text{or } DE = De + V \sec\theta.$$

But De may be found by resolving the forces at D in the direction De, which gives $De = Dd \cos\theta$. Therefore,

$$De = Dd \cos\theta + V \sec\theta.$$

We now find $V = 52.83$ and $Dd = 3.4$, whence $DE = +77.1$. In like manner $BC = +108.2$ and $FG = +49.3$.

The maximum stress in any lower counter, as EF, is equal to that in the corresponding member LM, and this may be found in the same way as for the main ties, backing the train toward the right. Thus $EF = LM = +3.44$ and $HG = HK = +24.7$.

The maximum stress for Bc and De is 18.4×0.7071 or $+13.0$ (since $Bc = Bb \cos\theta$, no matter what the stresses in Ba and BC may be). For Fg the stress cannot be greater than $+13.0$, even when the counter EF is strained, for in the latter case $Fg = +3.44 + 3.4 \times 0.707$.

Lastly, for long verticals, as Ee, the maximum stress is the

vertical component of the maximum stress in the tie DE. Thus,

$$Cc = -76.5, \quad Ee = -54.5 \quad \text{and} \quad Gg = -35.0.$$

This truss may also be used for through bridges, in which case the short sub-verticals become ties; Fig. 66 shows one form for such a truss.

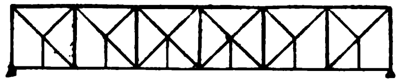

Fig. 66.

Prob. 82. For the through truss, in Fig. 66, let the span be 144 feet, the depth 24 feet, and the panel loads the same as above. Compute the maximum stresses in all the members.

ART. 55. THE POST TRUSS.

This form is intermediate in type between the Pratt and Warren trusses and is always with a double system of webbing, as shown in Fig. 67. The members Bb, Cc, etc., are struts and all the other diagonals are ties. The counters are shown in broken lines. On the upper chord the panels are equal, but in the lower chord AB is one-half a panel length. The struts hence have an inclination of one-half panel for the total depth and the ties an inclination of one and one-half panels. The stresses are computed by the same method as for the double system Pratt truss.

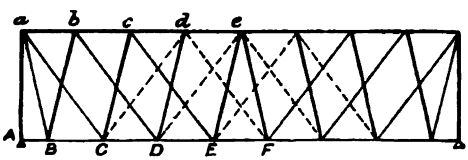

Fig. 67.

Let the truss in Fig. 67 be a deck span of 80 feet, the depth 20 feet, the dead load per linear foot per truss 500 pounds, and the live load 2 000 pounds, the live load being taken large enough to provide for full locomotive loading, and both dead and live loads being on the upper chord. The dead panel load is then 2.5 tons, and the live 10 tons.

It is unnecessary to again detail each step for computing the maximum and minimum stresses, but the following are the equations for a few of the pieces. It is seen that some ambiguity arises in the case of the counter ties, hence it is well to err on the safe side and compute them for live loads only.

$$\max CD = 12.5\left(4 \times \tfrac{5}{20} + 1\tfrac{1}{2} \times \tfrac{15}{20} + 1\tfrac{1}{2} \times \tfrac{5}{20}\right),$$
$$\max bc = 12.5\left(2 \times \tfrac{5}{20} + 1\tfrac{1}{2} \times \tfrac{15}{20} + 2 \times \tfrac{5}{20} + 1 \times \tfrac{15}{20}\right),$$
$$\max Dd = \left[2.5 + 10\left(\tfrac{5}{8} + \tfrac{3}{8} + \tfrac{1}{8}\right)\right] \times 1.0308,$$
$$\max De = 10\left(\tfrac{3}{8} + \tfrac{1}{8}\right) \times 1.25,$$
$$\max Ec = \left[1.25 + 10\left(\tfrac{4}{8} + \tfrac{2}{8}\right)\right] \times 1.25.$$

Thus we find for the chords,

	BC	*CD*	*DE*	*EF*	*ab*	*bc*	*cd*	*de*
Max	+ 12.5	+ 31.25	+ 43.75	+ 50.0	− 20.3	− 35.9	− 45.3	− 48.4
Min	+ 2.5	+ 6.25	+ 8.75	+ 12.5	− 4.1	− 7.2	− 9.1	− 9.7

for the ties,

	Ba	*Ca*	*Db*	*Ec*	*Fd*	*De*	*Cd*	*Bc*
Max	+ 25.8	+ 23.4	+ 17.2	+ 10.7	+ 6.2	+ 9.4	+ 4.7	+ 1.1
Min	+ 5.2	+ 4.7	+ 1.6	0?	0	0	0	0

and for the remaining members,

	Aa	*AB*	*Bb*	*Cc*	*Dd*	*Ee*
Maximum	− 61.25	0	− 25.8	− 19.3	− 14.2	− 9.0
Minimum	− 12.25	0	− 5.2	− 3.9	− 2.8	− 1.8

The Post truss is seen most frequently used for through bridges; it is now seldom built, although it possesses some theoretic advantages.

Prob. 83. Compute the stresses for Fig. 67 if it be a through bridge.

ART. 56. TRUE LIVE LOAD SHEARS.

In all the preceding examples, where a uniform live load per linear foot is used the largest live load shear at any section has been found by loading all the panel points on the right of the section with the live load, as in the first diagram of Fig. 68. Thus the first panel point on the right of the section has a full live load and the first one on the left has no live load. This assumption, however, is really an impossible one, for the live load is carried to the panel points by the stringers, and if any of it advance a distance x upon the panel a portion is transferred to the point on the left. The point n on the right cannot receive a full panel load until the train has advanced to the point $n + 1$, but then the point $n + 1$ receives a half panel load. The question now to be considered is: What is the value of the distance x in order that the shear in the panel may be the largest possible?

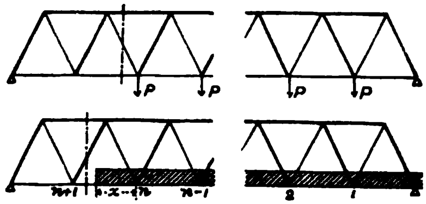

Fig. 68.

Let R be the left reaction and V the shear due to the live load on the panel between the points n and $n + 1$, these being the nth and $n + 1$th points from the right end of the bridge; let r_n and r_{n+1} be the portions (or reactions) of the load wx which are carried to the points n and $n + 1$. Then the shear upon the diagonals in this panel is $V = R - r_{n+1}$. If w be the live load per linear foot and p the length of the panel, the values of r_n and r_{n+1} are,

$$r_{n+1} = wx\frac{x}{2p} \qquad\qquad r_n = wx\left(1 - \frac{x}{2p}\right).$$

Hence if m be the number of panels in the truss,

$$R = r_n\frac{n}{m} + r_{n+1}\frac{n+1}{m} = wx\frac{n}{m} + \frac{wx^2}{2pm},$$

and accordingly the true shear due to the load wx is,

$$V = \frac{wxn}{m} + \frac{wx^2}{2pm} - \frac{wx^2}{2p},$$

The value of x which makes this a maximum is found by equating the derivative $\frac{dV}{dx}$ to zero, and is,

$$x = \frac{n}{m - 1} p,$$

which gives the position of the live load for the true largest shear in the $n + 1$th panel from the right end.

For instance, let the truss have 10 panels, then for the true maximum shear in the seventh panel from the right end we have $x = \frac{6}{9}p$, for the eighth panel $x = \frac{7}{9}p$, and for the tenth or last panel $x = \frac{9}{9}p$. Each panel, therefore, has a different position of the live load for true shear.

This result shows that the live load in the panel is $\frac{1}{m}$th of the total live load on the bridge; for the total live load is $nwp + \frac{n}{m - 1} wp$, which equals $\frac{mn}{m - 1} wp$, or m times the load on the panel.

The value of the true shear for any panel may be found by placing the live load in the proper position just deduced and then computing the panel loads and reactions. Thus, let the truss in Fig. 69 have 8 panels each 12 feet long; the live load per linear foot being 1.0 tons.

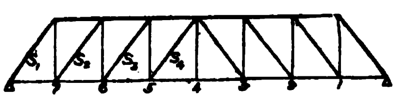

Fig. 69.

To find the shear for the member S_3 the live load extends from the right a distance $x = \frac{5}{7} \times 12$ beyond the point *5*. The live panel loads at *1*, *2*, *3* and *4* are each 12 tons; at *5* the panel load is $\frac{1}{2} \times 12 + \frac{5}{7} \times 12 \times 1.0 \times \frac{9}{14} = 11.51$ tons, and at *6* the panel load is $\frac{5}{7} \times 12 \times 1.0 \times \frac{5}{14} = 3.06$ tons. The left reaction due

to these live loads is then found to be,

$$R = 12\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8}\right) + 11.51 \times \tfrac{5}{8} + 3.06 \times \tfrac{6}{8} = 24.49,$$

and the shear for the diagonal S_3 is,

$$V_3 = 24.49 - 3.06 = +\ 21.43 \text{ tons.}$$

By the usual method of taking a full panel load at *5* and none at *6*, we have,

$$V_3 = R = 12\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8}\right) = +\ 22.5 \text{ tons.}$$

The usual method hence gives the shears too great and its error s on the safe side.

The following is a comparison for the above example of the live load shears found by the usual method and by the stricter method here explained:

	V_1	V_2	V_3	V_4
Usual method	+42.0	+31.5	+22.5	+15.0
Strict method	+42.0	+30.86	+21.43	+13.71
Difference	0.0	0.64	1.07	1.29

	V_5	V_6	V_7	V_8
Usual method	+9.0	+4.5	+1.5	0.0
Strict method	+7.71	+3.43	+0.86	0.0
Difference	1.29	1.07	0.64	0.0

It is seen that the difference is least at the ends of the truss and greatest near the middle. Inasmuch as the differences are all small and the usual method errs on the safe side, the latter is generally employed, and will hereafter in this Chapter be used unless otherwise specified.

In the case of a preceding locomotive load a similar discrepancy exists between the true shears and those found by the usual method. As, however, it is a matter of judgment in stating this load and the number of panels it is to cover, it is not advisable to introduce refinements of calculation which are complicated and do not render the final results more reliable. The above demonstration, moreover, applies only to a truss with a single system of webbing, and is wholly inapplicable to a double system truss.

Prob. 84. Compute the true shears and diagonal stresses for the Warren truss of Art. 34.

ART. 57. ONE CONCENTRATED EXCESS LOAD.

It is sometimes specified that the trusses shall be computed for a uniform train load per linear foot upon which at any point a single excess load may be placed. Fig. 70 symbolizes this kind of loading. The excess load P is usually taken as the difference between one locomotive panel load and one train panel load. If the uniform train load per linear foot be taken at 3 000 pounds the value of the excess load P should range from 50 000 to 60 000 pounds, depending usually on the panel length and other circumstances.

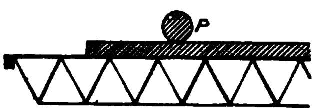

Fig. 70.

To compute the maximum stresses in a truss under such loads we can first find the stresses due to the dead and train loads by the methods already given; to these are to be added the largest stresses due to the load P. It is, therefore, necessary to inquire where P should be placed to produce the largest shear and moment at any section.

The largest positive shear at any section caused by a single load P will be caused when that load is on the right of the section and as near to it as possible. For, the nearer the load to the section the greater the left reaction R (see Art. 32). The locomotive should hence precede the train to give the maximum

stress in the web members, and the load P should be put at the panel point on the right of the section. A full train load will be taken also at this panel point, as by the usual method.

The largest bending moment, and hence the largest stress for any chord member, will be caused by P when it is placed at the center of moments for that member. For then the chord stress is $\frac{Rc}{d}$, where c is the lever arm of the reaction R and d is the lever arm of the stress. But R will be the greatest when the load is at the section through the center of moments (see Art. 30). This case will occur when the locomotive is pushing and pulling cars at the same time.

To illustrate, take the Howe truss in Fig. 69. Let the span be 130 feet, the depth 20 feet, the dead load 1 200 pounds, the live train load 3 000, both per linear foot, and the excess load P 54 000 pounds. It is required to find the maximum stresses in all members, for a single track through bridge. First, the panel loads are found to be, dead = 4.5 tons, train = 11.25 tons and excess = 13.5 tons.

For any diagonal, as S_3, the train covers the points *1*, *2*, *3*, *4* and *5*, and the excess load is put at *5*. The shear in S_3 then is,

$$V_3 = 3\tfrac{1}{2} \times 4.5 + \frac{11.25}{8}(1 + 2 + 3 + 4 + 5) + 13.5 \times \tfrac{5}{8} - 2 \times 4.5 = +36.28,$$

and the maximum stress is,

$$S_3 = -36.28 \times \sec\theta = -36.28 \times 1.25 = -45.4 \text{ tons}.$$

The shear 36.3 tons is the maximum tensile stress in the vertical tie to the right of S_3.

For the upper chord above S_3, the train covers the whole bridge and the excess load is put at the point *6*. Then the left reaction is,

$$R = 3\tfrac{1}{2}(4.5 + 11.25) + 13.5 \times \tfrac{5}{8} = 65.25 \text{ tons},$$

and the equation of moments for the member is,

$$62.25 \times 30 - 27 \times 15 + S \times 20 = 0,$$

whence the maximum compression is 77.6 tons.

In this manner we find the following maximum stresses: For the vertical ties beginning at the left end, 66.9, 50.9, 36.3 and 23.1 tons; for the main struts, 83.7, 63.6, 45.4 and 28.8 tons; for the lower chord, 50.2, 86.1, 107.6 and 114.8 tons; for the counters, 14.0 tons and 1.1 tons.

Prob. 85. Find the maximum stresses in a through Pratt truss for a single track bridge, the span being 176 feet, panel length 16 feet, dead load 1 500 pounds, live load 3 000 pounds, both per linear foot, and an excess load of 56 000 pounds.

ART. 58. TWO CONCENTRATED EXCESS LOADS.

As the effect of one locomotive may be approximately represented by a single excess load, so the effect of two coupled locomotives may be represented by two excess loads placed 50 feet apart. As before, we take the weight of the train at about 3 000 pounds per linear foot, and each excess load at from 50 000 to 60 000 pounds.

The largest positive shear at any section due to two loads always 50 feet apart, will occur when both loads are on the right of the section and one of them as near to it as possible. For, in this case the shear V is the reaction R, which is the greater the nearer the loads to the left abutment, but if the front load pass the section, V is decreased by P.

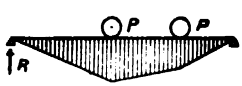

Fig. 71.

The largest bending moment, and hence the largest chord stress at any section, due to two equal loads 50 feet apart, occurs when one of them is at the center of moments for the given section and the other is on that side of the section which brings it nearest to the middle of the bridge. Accordingly for a section

in the left hand half of the truss the second load is always to the right of the first. The proof of this will be evident by considering the diagram of moments in Fig. 71.

To simplify the computation it is often customary to take the distance between the two loads less or greater than 50 feet, in order to make both loads come upon panel points. Thus for 12, 13, 15 and 17 foot panels the distances would be 48, 52, 45 and 51 feet respectively.

For example, we take a through Pratt truss of 162 feet span for a double track bridge, having 9 panels on the lower chord each 18 feet long, and 24 feet deep. The dead load is 1 500 pounds per foot, one-third of which is to be taken on the upper chord and two-thirds on the lower chord. The train load is 3 100 pounds per foot and there are two excess loads, 54 feet apart, each weighing 50 000 pounds. It is required to compute the maximum stresses in all members.

We first find the panel loads to be: Dead on upper chord 4.5 short tons; dead on lower chord 9.0 tons; train 27.9 tons; and each excess 24 tons. A full dead panel load should be taken at the first apex of the upper chord on account of the heavy portal bracing there.

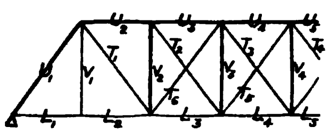

Fig. 72.

To find the maximum stress for the lower chord L_3, the dead and train loads cover the whole truss, one excess load is placed at the left end of L_3 and the other three panels to the right. The reaction is,

$$R = 4\,(4.5 + 9.0 + 27.9) + 25\,(\tfrac{7}{9} + \tfrac{4}{9}) = 196.155 \text{ tons},$$

and the equation of moments is,

$$L_3 \times 24 = 196.155 \times 36 - 41.4 \times 18,$$

whence $L_3 = 263.2$ tons, the maximum tension.

To find the maximum stress in the diagonal T_2, the dead load covers the whole truss, the train load is at every panel point on the right of a section cutting T_2, one excess load is at the foot of T_2 and the other three panels to the right. The reaction due to these loads is,

$$R = 4(4.5 + 9.0) + \frac{27.9}{9}(6 + 5 + 4 + 3 + 2 + 1)$$
$$+ \frac{25.0}{}(6 + 3) = 144.1 \text{ tons.}$$

Hence the stress in T_2 is,

$$T_2 = (144.1 - 2 \times 13.5)\frac{\sqrt{18^2 + 24^2}}{24} = + 146.4 \text{ tons.}$$

The maximum stress in V_2 is the same as the maximum shear, since sec $\theta = 1$, and is,

$$V_2 = 144.1 - 2 \times 9.0 - 1 \times 4.5 = 121.6 \text{ tons compression.}$$

The minimum stresses will be found for the chords by using dead load only, and for the main ties by placing the live load in the position to give minimum shear. The final values are as follows:

For the chords,

	L_1 and L_2	U_2 and L_3	U_3 and L_4	U_4 and L_5
Maximum	151.3	255.7	335.7	368.8
Minimum	40.5	70.9	91.1	101.2

For the end post and verticals,

	U_1	V_1	V_2	V_3	V_4
Maximum	− 252.2	+ 61.9	− 121.6	− 83.9	− 49.4
Minimum	− 67.5	+ 9.0	− 31.5	− 18.0	− 4.5

For the main and counter ties,

	T_1	T_2	T_3	T_4	T_5	T_6
Maximum	+ 197.3	+ 146.4	+ 99.3	+ 56.1	+ 16.5	0
Minimum	+ 43.4	+ 15.2	0	0	0	0

To the above chord stresses are to be added and subtracted the wind stresses, if so specified.

Prob. 86. Compute the maximum and minimum stresses for a through triangular Warren truss of 192 feet span for a double track, having 16 panels, each 12 feet long, and 24 feet deep; the dead load being 1 600 pounds, the train load 3 000 pounds, both per linear foot of single track, and two excess loads 48 feet apart, each of 54 000 pounds.

ART. 59. LOCOMOTIVE WHEEL LOADS.

The live load which is now in most general use for railroad bridges is a uniform train load of about 3 000 pounds per linear foot of single track, preceded by two coupled locomotives, the actual wheel loads being taken. The first of the following diagrams represents the two typical consolidation locomotives and the second the two typical passenger locomotives, specified by the Pennsylvania Railroad. The numbers between the wheels

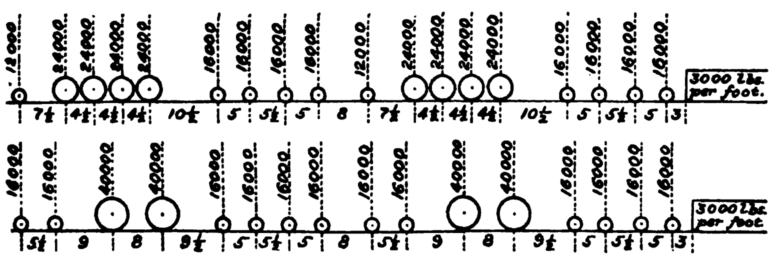

Fig. 73.

show their distances apart in feet, and those above the wheels show their weights in pounds for both rails of a single track. It is seen that two coupled locomotives with their tenders cover about 105 feet in length, so that for bridges of this span or less no uniform live load is used.

A typical locomotive is one which does not actually exist, but which is supposed to give as great or greater stresses than any in use, or any which are likely to be built for some years to come. It is often specified that actual locomotives shall be used for the computation of stresses, but as the distances apart of the wheels are given in inches and fractions of inches, it is evident that a typical locomotive simplifies the numerical work. Of course a great variety of patterns of locomotives are in use and any of the heavier ones are liable to be specified by railroad companies to be used for stress calculations.

Prob. 87. Find the uniform load per linear foot which will cause the same maximum bending moment in a beam of 20 feet span as the drivers of the above passenger locomotive.

Ans. 5 120 pounds.

ART. 60. SHEARS FROM WHEEL LOADS.

For determining the maximum stress in a web member the train comes upon the bridge from the right until the head of the front locomotive stands near the panel point nearest to the right of the member. It will now be proved that the largest possible shear due to the live load occurs when the load on the panel is $\frac{1}{m}$th of the total live load on the bridge, m being the number of panels in the loaded chord of the truss. The demonstration applies to a truss with a single system of webbing, where the loads are carried by stringers to the panel points.

Let Fig. 74 represent a passenger locomotive with tender and train on a bridge; let x be the distance of the front of the train from the right abutment, and a, b, ... h, the distances of the loads, P_1, P_2, ... P_8 from the front of the train. Then, if the train load

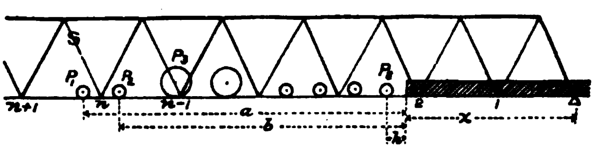

Fig. 74.

per linear foot be w, the total live load on the bridge will be,

$$W = P_1 + P_2 + \ldots + P_8 + wx.$$

Let p be the panel length and m the number of panels, the span is then mp, and the left reaction is,

$$R = P_1 \frac{a + x}{mp} + P_2 \frac{b + x}{mp} + \ldots . + P_8 \frac{h + x}{mp} + wx \frac{\frac{1}{2}x}{mp}.$$

Now, if the load P_1 be at a distance y beyond the nth panel point from the right end, the shear for the member S will be this reaction minus that part of P_1 carried to the point $n + 1$ by the stringers, or

$$V = (P_1 a + P_2 b + \ldots + P_8 h) \frac{1}{mp} + (P_1 + P_2 + \ldots P_8) \frac{x}{mp} + \frac{wx^2}{2mp} - P_1 \frac{y}{p}.$$

Now, since $x + a = np + y$ we may insert for y its value in terms of x. To abbreviate, let P denote the entire locomotive load $P_1 + P_2 + \ldots + P_8$, and g denote the distance of its center of gravity from the front of the train; then

$$V = P \frac{g}{mp} + P \frac{x}{mp} + \frac{wx^2}{2mp} - P_1 \frac{x + a - np}{p},$$

is the true live load shear for the member S.

If the train advance the distance dx, the shear V receives the increment,

$$dV = \left(\frac{P}{mp} + \frac{wx}{mp} - \frac{P_1}{p}\right) dx,$$

and by equating this to zero we may find the value of x which renders V a maximum, providing that P_1 thereby remains on the panel and that no other loads come upon it. Instead, however, of finding x we may equate the derivative to zero, and write

$$\frac{P + wx}{m} - P_1 = 0,$$

or in another form, since $P + wx = W$, we have the rule,

$$P_1 = \frac{1}{m} W;$$

that is, the shear is the largest when the load on the panel equals $\frac{1}{m}$th of the total live load on the bridge. This is the same result as found for uniform live load in Art. 56.

Although in the above demonstration only one wheel has been taken on the panel the reasoning is general and the same conclusion results, whatever be the number of loads supposed to be there. In order to satisfy the condition one of the loads in general must come at n, so that, if necessary, a part of it together with the preceding loads on the panel may be $\frac{1}{m}$th of the total live load.

For example, let each driver in Fig. 74 be 20 tons, each of the other wheels 8 tons, and there be no train following. The total load W is 88 tons. If the truss have 12 panels, the first wheel must be put at n, since $8 > \frac{88}{12}$. If it have 6 panels the second wheel P_2 must come at n, since $8 < \frac{88}{6}$ and $8 + 8 > \frac{88}{6}$.

This rule is easy of application, and the load being put in proper position the true shear is readily found. It is, however, often specified that the shear shall be computed by placing the first driver at the panel point, neglecting the part of the load transferred to the preceding panel point. The error of this method is almost always on the safe side.

To illustrate, let the loads in Fig. 74 be 20 tons for the drivers, 8 tons for each of the other wheels and 1.5 tons per foot for the train. Let the number of panels be 7, the panel length 10 feet, and let it be required to find the shear for the panel where $n = 5$. By the above rule, if the second pilot wheel be at n, we find $W = 88 + 1.5 \times 5 = 95.5$, one-seventh of which is 13.6; as $8 + 8$ is greater than 13.6, this is the correct position for true

largest shear. The reaction for the loads in this position is,

$$R = \frac{8}{70}(55\tfrac{1}{2} + 50 + 23\tfrac{1}{2} + 18\tfrac{1}{2} + 13 + 8) + \frac{20}{70}(41 + 33) + \frac{1.5 \times 5^2}{2 \times 70} = 40.7 \text{ tons},$$

and the true shear hence is,

$$V = 40.7 - 8 \times \frac{5.5}{10} = 36.3 \text{ tons}.$$

By the practical rule we place the first driver at *n*, and find the reaction to be 53.8 tons, which is also the shear. It hence appears that the practical rule often errs largely in excess. If it is merely specified that the 'maximum stress' shall be found, a bridge company usually prefers to use the exact method as less material is thereby required for webbing.

Prob. 88. A truss of 80 feet span has 8 panels, each 10 feet long. Find the true live load shear for one of the panels caused by a single typical consolidation locomotive and tender.

ART. 61. MOMENTS FROM WHEEL LOADS.

For finding the stress in any chord member the live load must be so placed that the bending moment with respect to the center of moments for the given member is the largest possible. Thus let it be required to find the position of the loads in Fig. 74, which will give for the point *n* the largest bending moment, and hence the largest stress for the upper chord above that point.

Let P' be the part of the load on the left of *n*, and g' the distance of its center of gravity from *n*. Let $n'p$ be the distance from the left abutment to the point *n*, and the other notation as in the last Article. Then the bending moment at *n* is,

$$M = R.n'p - P'g'.$$

Inserting the value of R this becomes,

$$M = \left(P\frac{g}{mp} + P\frac{x}{mp} + \frac{wx^2}{2mp}\right)n'p - P'g'.$$

Now, if the loads advance a distance dx, x and g' increase to $x + dx$ and $g' + dx$, so that the increment of M is,

$$dM = \left(\frac{Pdx}{mp} + \frac{wxdx}{mp}\right)n'p - P'dx,$$

and equating this to zero, we have the condition for a maximum, namely,

$$P' = \frac{n'}{m}(P + wx), \qquad \text{or} \qquad P' = \frac{n'}{m}W,$$

that is, the load on the left of the section is $\frac{n'}{m}$th of the total live load on the bridge. Hence the load on the right of the section is $\frac{n}{m}W$.

Since n' is to m as the distance on the left of the section is to the span, this condition shows that the loads are to be so placed that the weights on the two segments of the span are proportional to the lengths of those segments. To satisfy this condition one of the wheels must in general be placed at the center of moments.

For example, let it be required to find the greatest stress in the upper chord of a Warren truss whose span is 70 feet, panel length 10 feet, depth 12 feet, for the panel where $n = 4$, due to the locomotive in Fig. 73, without train load. Here $P = W = 88$ tons, $m = 7$ and $n' = 3$; then P' must equal $\frac{3}{7} \times 88$ or $37\frac{5}{7}$ tons; to insure this the loads P_1, P_2 and P_3 must be on the left of n, and P_4 be at n, since $8 + 8 + 20 < 37\frac{5}{7}$. The reaction is,

$$R = \frac{8}{70}(62.5 + 57 + 30.5 + 25.5 + 20 + 15) + \frac{20}{70}(40 + 48) = 48.91 \text{ tons.}$$

The bending moment then is,

$$M = 48.91 \times 30 - 8(22.5 + 17) - 20 \times 8 = 991.3 \text{ tons-feet,}$$

and finally, the stress for the upper chord is one-twelfth of 991.3 or 82.6 tons compression.

If the locomotive be followed by a uniform load or by a second locomotive, of course all the live load on the bridge must be included in the weight W, and in the determination of the reaction.

The above demonstration applies to the unloaded chord of any single system truss, and also to the loaded chord for the Pratt, Howe and other types where the panel points of the upper chord are vertically above those of the lower chord. It does not apply to the loaded chord of trusses like the Warren, where all the web members are inclined. The rule for this case, which may be deduced by reasoning similar to the above, is the following Let P' be the load on the left of the given panel, Q the load on the panel, and W the total load; let l be the length of the span and l' the distance of the center of moments from the left support; then the load is to be so placed that

$$P' + \tfrac{1}{2} Q = \frac{l'}{l} W.$$

This supposes that the center of moments is vertically over or under the center of the given panel, as is the case in all usual constructions. If the center of moments is horizontally distant, the amount q from the left end of the given panel, the rule is,

$$P' + \frac{q}{p} Q = \frac{l'}{l} W,$$

where p is the panel length. For the Howe and the Pratt types q equals zero, and this becomes,

$$P' = \frac{l'}{l} W = \frac{n'}{m} W,$$

or the same as before found for the unloaded chord.

Prob. 89. A deck Warren truss of 80 feet span has 8 panels, each 10 feet deep. Find the greatest chord stress for a panel of the upper chord due to a single typical consolidation locomotive and tender.

ART. 62. TABULATION FOR LOCOMOTIVE WHEELS.

The numerical labor of computation of stresses is materially lessened by tabulating for each locomotive the various moments in the manner shown in Fig. 75. Only one passenger locomotive with its tender is here used, but the same method applies for two locomotives. The figure shows at a glance the weights and

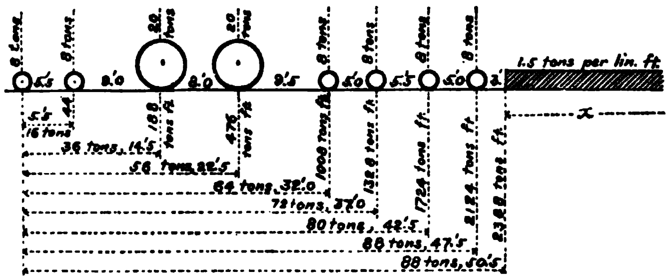

Fig. 75.

distances apart of the wheels; below each wheel on the horizontal line is shown the weight of that wheel together with the preceding ones and its distance from the front wheel, while on the vertical line is given the moment of the preceding wheels with reference to that wheel. Thus, for the second driver we have 56 tons for the weight of it and the preceding wheels, 22.5 feet for its distance from the front wheel, and 476 tons-feet for the moment of the preceding loads.

This diagram may be used for finding both reactions, shears and moments, and it will be found convenient to draw a skeleton outline of the truss to the same scale to be placed directly above it in the proper position for maximum stress in each member. As in Art. 60, let P_1, P_2, ... P_8 denote the wheel loads, at distances a, b, ... h from the head of the uniform train load, and let x be the distance from this point to the right abutment. Then, if l be the length of the span, and w the train load per linear foot, the left reaction is,

$$R = P_1 \frac{a + x}{l} + P_2 \frac{b + x}{l} + \dots + P_8 \frac{\;+ x}{l} + \frac{wx^2}{2l}.$$

This may be written, if P denote the sum $P_1 + P_2 + \ldots . P_8$,

$$R = \frac{1}{l}(P_1a + P_2b + \ldots . P_8h + Px + \tfrac{1}{2}wx)$$

$$= \frac{1}{l}(2\ 388 + 88x + \tfrac{1}{2}wx^2).$$

Again, if the right abutment be at a point midway between the first and second wheels of the tender, we take the quantities 1 008 and 64 from the diagram and have at once the reaction,

$$R = \frac{1}{l}(1\ 008 + 64 \times 2.5).$$

Again, let us suppose that the last wheel of the tender must be put at a panel point l' feet from the left end to give a maximum chord stress. Then the reaction is found, the quantity 2 124 taken from the diagram, and the maximum bending moment is $Rl' - 2\ 124$. The example in the next Article will further show the great convenience of tabulating the wheel moments as in Fig. 75. Particularly is this the case when the wheel distances are given in inches and fractions of inches.

Prob. 90. If the span be 100 feet and the first wheel be 15 feet from the left abutment, find the reaction for the loads shown in Fig. 75. Also when the first wheel is 50 feet from the left abutment. Also the moment for the center of the span, the value of x being 42 feet.

ART. 63. STRESSES FROM LOCOMOTIVE AND TRAIN LOADS.

It is required to compute the maximum and minimum stresses for a double track through Pratt truss of 140 feet span having 7 panels, each 20 feet long, and 24 feet in depth. The dead load per linear foot is 1 400 pounds or 0.7 tons, the live load a passenger locomotive and tender as in Fig. 75, followed by a uniform train load of 3 000 pounds, or 1.5 tons per linear foot.

A skeleton outline of the truss, on a scale of about 20 feet to an inch is first made, and near the edge of another sheet Fig. 75 is drawn to the same scale, so that it may be placed at any point of the truss diagram. For all the inclined members the value of sec θ is 1.302.

It will be convenient to compute separately the stresses due to dead and live loads. No explanation will be necessary for the dead load stresses in view of the examples already given in this chapter and the last. The stresses for the chords and diagonals will be the same whether all dead load be taken on the lower chord or a part on the upper, but those for the vertical posts will be greater if a part be taken on the upper chord. In the values given below 400 of the 1 400 pounds per linear foot is regarded as on the upper chord.

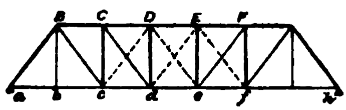

Fig. 76.

To find the position for maximum shear in Cc and Cd caused by the live load we lay the diagram of Fig. 75, so that the first pilot wheel is at d; the value of x for the uniform train load is $4 \times 20 - 50.5 = 29.5$ feet and the total live load on the truss is $88 + 29.5 \times 1.5 = 132.25$ tons; since $8 < \frac{132}{7}$ this is not the correct position. If the second wheel be at d the load on the truss is greater than 132 tons and 16 is also less than one-seventh of this. With the first driver at d the value of x is $29.5 + 14.5 = 44$ feet and the total live load is $88 + 44 \times 1.5 = 154$ tons, one-seventh of which is 22 tons; $8 + 8 + 20$ is greater than this, and hence for maximum shear in the panel the first driver comes at d.

With the live load in this position, the value of the left reaction is,

$$R = \frac{1}{140}(2\,388 + 88 \times 44 + 1.5 \times \tfrac{1}{2} \times 44^2) = 55.09 \text{ tons},$$

and the true maximum shear is equal to this reaction minus the

part of the load on *cd* which is carried to *c*, or,

$$V = 55.09 - \frac{8}{20}(14.5 + 9) = 45.69 \text{ tons.}$$

This shear is the live load stress on *Cc*, and multiplying it by the secant 1.302 we have 59.5 tons for the live load stress on *Cd*.

In the same manner the maximum live load stress for each of the other web members is found. The first driver will come at the panel point in all cases except for *Ef*, where the second pilot wheel should be placed at *f*.

To find the position for maximum moment in *cd* we lay the wheel diagram on the truss, so that the first tender wheel comes at *c*; the value of x is then 81.5 feet and the total live load is $88 + 1.5 \times 81.5 = 210.25$ tons; the part of this on the left of *c* is 56 tons, and $\frac{ac}{ah} = \frac{n'}{m} = \frac{2}{7}$; as 56 is almost equal to $\frac{2}{7}$ of 210.25, this is the correct position for maximum moment. For this load the reaction is,

$$R = \frac{1}{140}(2\,388 + 88 \times 81.5 + 1.5 \times \tfrac{1}{2} \times 81.5^2) = 103.87 \text{ tons,}$$

and the bending moment for *c* is,

$$M = 103.87 \times 40 - 1\,008 = 3\,146.8 \text{ tons-feet.}$$

Dividing this by the depth, 24 feet, we have 131.1 tons as the live load stress in *cd*.

Similarly for the other chord members we proceed. For *bc* the first driver stands at *b*, and for *CD* the last tender wheel stands at *d*. After a little practice the student will be able to place the live load in proper position at the first trial.

The following are the dead and live load stresses for this example, from which in the usual manner the final maximum and minimum stresses may be obtained :

For the chords and end post,

	ab and *bc*	*BC* and *cd*	*CD* and *de*	*Ba*
Dead stress	35.0	58.3	70.0	— 54.7
Live stress	81.9	131.1	154.6	— 127.9

For the webbing,

	Bb	*Cc*	*Dd*	*Bc*	*Cd*	*De*	*Ef*
Dead stress	+ 10.0	— 18.0	— 4.0	+ 36.5	+ 18.2	0	0
Live stress	+ 38.8	— 45.7	— 25.8	+ 90.9	+ 59.5	+ 33.6	+ 15.4

It is to be observed that for the panels near the end of the truss a greater chord stress may sometimes be obtained by allowing one or both pilot wheels to pass off the bridge, although it is not the case in this example; thus, for *ab*, if the driver stands at *b*, the condition of the rule in Art. 61 is fulfilled and the live load stress is 81.7 tons, but by putting the second driver at *b* the first pilot wheel passes off the bridge, the condition is also fulfilled, and we have the smaller stress 81.1 tons. The same remark applies to the counter *Ef*, where the condition of Art. 60 is satisfied both by the first driver and second pilot wheel at *f*.

The maximum live load stress for the sub-vertical *Bb* is found by placing the drivers so as to bring the greatest load at *b*. Since the live load stress for *Ef* or *Dc* is less than the dead load stress in *Cd*, no counters are theoretically needed except for the middle panel.

If the specifications require the truss to be computed for two coupled locomotives, followed by a uniform train load, as is generally the case, a tabulation like Fig. 75 should be made for the wheels of both locomotives, and then the stress computations are pursued exactly as here illustrated.

Prob. 91. Make a tabulation of moments, like Fig. 75, for two coupled typical passenger locomotives.

Prob. 92. A through Pratt truss, like Fig. 76, has the same dimensions and dead load as in the above example. Compute the live load stresses caused by two coupled typical passenger locomotives, followed by a uniform train load of 3 000 pounds per linear foot. Find also the final maximum and minimum stresses for each member.

ART. 64. REMARKS ON DOUBLE SYSTEMS.

The preceding investigations for the largest shear and moment due to concentrated weights and wheel loads have in general been limited to a single system of web triangulation. We now point out the modifications and assumptions necessary when a truss has a double system of webbing.

For the case of a single excess load (Art. 57) each web system may be separately computed, keeping the load at the head of the train and regarding the stresses in each system as due only to the loads which come upon it. For the chords the excess load should only be placed upon one of the systems, and preferably upon that which will render the chord stress the greater. Thus, for the deck truss in Fig. 77, if it be required to find the stress for *CD* the single load should be taken at *E*; and in general it should be placed near that end of the member which is nearest the middle of the truss.

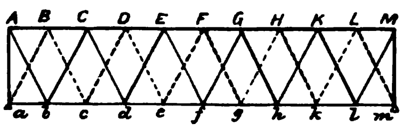

Fig. 77.

For two excess loads (Art. 58), the same observations apply, but the first load may be on one system and the second on the other if the data so require. Thus the maximum stress in *CD* might be caused by one load at *C* and the other at *F*.

For the shear and moment due to wheel loads the investigations given and the rules deduced, in Arts. 60 and 61, apply only to single systems. Here it will be best to use the practical rule for the shears, placing the first driver at the panel point and neglecting the portion transferred to the other system by the preceding floor beam. Each system is then to be regarded as acting independently of the other and carrying only the loads which rest upon it. As in Art. 50 the systems are to be divided symmetrically to the middle of the truss for dead load and unsymmetrically for the live load. For the chords it will probably be best for short spans to place the locomotive in the middle of the truss, let it be both preceded and followed by the train, and then compute the stresses for this position, regarding each system of diagonals as carrying the loads which come upon it. The rule of Art. 61 might be regarded as approximately applicable to double systems, taking $\frac{n'}{m}$ for the panel point nearest the middle, but this in general will require much labor in finding the panel loads for the separate systems.

Methods of replacing wheel loads by equivalent uniform loads are also in use, but these have the disadvantage that such loads do not give all the maximum moments correctly and that it gives the shears generally too small.

Double systems are now not regarded with so much favor as formerly, and it has been found possible to use single systems for the longest spans by the use of sub-verticals to prevent long panel lengths. The indications are that single intersection trusses are the most economical, and certainly they are theoretically the most scientific, since the true static stresses may be computed without ambiguity.

Prob. 93. Let the double system deck Warren truss in Fig. 77 have 10 panels, each 12 feet long and 12 feet deep. Find the maximum stresses due to a dead load of 1 200 pounds, a train load of 3 000 pounds, both per linear foot, and two excess loads 48 feet apart each of 54 000 pounds.

ART. 65. EXAMPLE OF A DOUBLE SYSTEM TRUSS.

Let the double system deck Warren truss, shown in Fig. 77, have 10 panels, each 12 feet long and 12 feet deep, the dead load being 1 200 pounds per linear foot, of which one-third is to be taken on the lower chord. The live load is a typical passenger locomotive, as in Fig. 75, followed by a uniform train load of 3 000 pounds per linear foot. It is required to compute the dead and live load stresses in all the members.

By the methods of Arts. 27 and 50 the dead load stresses are found, the values being as stated below.

Diagrams of the truss and of Fig. 75 to the same scale are drawn as before explained. Without making and using these diagrams it will be difficult for the student to follow the numerical work below.

To find the live load stresses in any web member we use the practical rule of Art. 64, and place the front driver at the panel points, neglecting the part carried by the stringers to the other system. For instance, to find the shear for *De* and *Fe* the first driver is put at *F;* then the panel loads at *F*, *H* and *L* will be,

$$F = 8 \times \tfrac{8}{12} + 20 + 20 \times \tfrac{4}{12} = 28.67 \text{ tons},$$

$$H = 8\left(\tfrac{5.5}{12} + \tfrac{10.5}{12} + \tfrac{8}{12} + \tfrac{8}{12}\right) = 18.0 \text{ tons},$$

$$L = 1.5 \times 12 = 18.0 \text{ tons}.$$

The reaction due to these is,

$$R = 28.67 \times \tfrac{5}{10} + 18 \times \tfrac{3}{10} + 18 \times \tfrac{1}{10} = 21.58 \text{ tons},$$

which is the shear for *De* and *Fe ;* multiplying this by the secant 1.4142, we have 30.5 tons for the live load stress in these members.

In like manner to find the stress for *Ef* and *Gf* we place the front driver at *G*, and have the panel loads *G* and *K* as 28.7 and 18.0 tons ; then the reaction is 10.41 tons, and this is the shear which from the live load stress is 14.7 tons, which of course is tension in *Ef* and compression in *Gf*.

To find the chord stresses we place the locomotive so that its rear driver rests at *F*, and both in front and rear have the train load of 1.5 tons per linear foot. The panel loads for the two systems then are,

$$B = 18.0,\quad D = 18.3,\quad F = 28.3,\quad H = 18.3,\quad L = 18.0,$$
$$C = 18.0,\quad E = 19.0,\quad G = 15.3,\quad K = 17.0,$$

from which the left reaction for the first system is 50.45 tons and for the second 35.52 tons. Now, to find the stress for *EF*, we have,

$$S' = 50.45 \times 4 - 18.0 \times 3 - 18.3 \times 1 = 129.5 \text{ tons},$$
$$S'' = 35.52 \times 5 - 18.0 \times 3 - 19.0 \times 1 = 104.6 \text{ tons};$$

whence $EF = 129.5 + 104.6 = 234.1$ tons compression.

In this manner are computed the following values, from which by addition the final stresses are found:

For the diagonals,

	Ab	*Bc*	*Cd*	*De*	*Ef*
Dead stress	+ 22.1	+ 17.0	+ 11.9	+ 6.8	+ 1.7
Live stress	+ 63.0	+ 51.3	+ 39.6	+ 30.5	+ 21.3
	− 0.0	− 4.1	− 8.1	− 14.7	− 21.3

	Fe	*Ed*	*Dc*	*Cb*	*Ba*
Dead stress	− 3.4	− 8.5	− 13.6	− 18.7	− 22.5
Live stress	+ 14.7	+ 8.1	+ 4.1	+ 0.0	+ 0.0
	− 30.5	− 39.6	− 51.3	− 63.0	− 77.2

For the chords,

	AB	*BC*	*CD*	*DE*	*EF*
Dead stress	− 15.6	− 44.4	− 66.0	− 80.4	− 87.6
Live stress	− 35.5	− 118.4	− 171.5	− 218.1	− 234.1

	ab	*bc*	*cd*	*de*	*ef*
Dead stress	+ 16.8	+ 45.6	+ 67.2	+ 81.6	+ 88.8
Live stress	+ 50.5	+ 121.5	+ 186.4	+ 216.5	+ 244.8

Prob. 94. Check the values above given for several of the web and chord members and find the stresses for the end post *Aa*.

ART. 66. TRIPLE AND QUADRUPLE SYSTEMS.

Triple systems are rarely used in modern practice and the few formerly built were of the Whipple type as shown in Fig. 78. The computation of these is made in the same manner as for the double system, each set of diagonals being supposed to act independently of the other two.

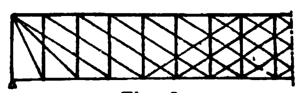

Fig. 78.

The lattice girder, or quadruple Warren truss, as shown in Fig. 57, is more frequently built. To this the methods above explained for double systems likewise apply and the computation of stresses is effected without difficulty and also without ambiguity, if only uniform live load be used for the chords. For instance, let such a through truss have a span of 204 feet, a depth of 25.5 feet and 16 panels each of 12.75 feet, the dead load per linear foot per truss being 0.4 tons and the live load 1.0 ton. (Let the student draw the figure.)

To find the maximum chord stresses the live load covers the whole truss and the full panel load is 17.85 tons; the systems are divided symmetrically to the center of the truss in an analogous manner to Fig. 61. Then for the stress in the fourth panel of the lower chord we have by increments,

$$S = 17.85 \left(1\tfrac{1}{2} + 2 + 1 + 2 + 1 + 2 + 1\right) \times 1 = 134 \text{ tons.}$$

Again, for the third diagonal tie, the dead load acts in the symmetrically divided systems and the live in the unsymmetrical;

the dead panel load is 5.1 tons and the live 12.75 tons; then

$$S = [5.1 \times 2 + 12.75 (\tfrac{13}{16} + \tfrac{9}{16} + \tfrac{5}{16} + \tfrac{1}{16})] \times 1.4142 = 46 \text{ tons.}$$

Thus are computed the stresses. The values for all members may be seen in JOHNSON's Cyclopedia, Vol. II., p. 155.

Prob. 95. Find the maximum stresses for the the triple truss in Fig 78, the span being 240 feet, the number of panels 20, the panel length 12 feet, the depth 36 feet, the dead load per linear foot 0.5 tons and the live 1.5 tons.

ART. 67. UNSYMMETRICAL TRUSSES.

The trusses of a skew bridge should be so placed that the floor beams may be at right angles to the trusses. This often brings the first panel point of one truss directly opposite to the second or third panel point of the other truss, so that the trusses are alike and the two halves of each are symmetrical. When the degree of skew will not allow this to be done unsymmetrical trusses are built.

Fig. 79 represents the side elevation of an unsymmetrical through Pratt truss, together with a plan of the lower chords,

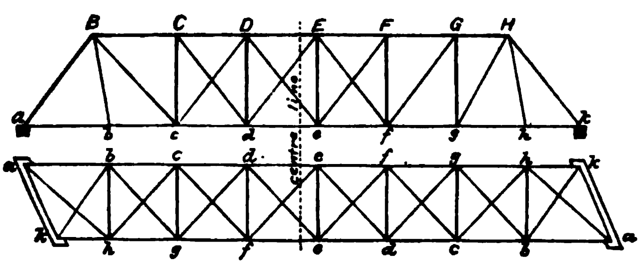

Fig. 79.

floor beams and lateral bracing. In the lower chord the panels between *b* and *h* are all equal as are also those in the upper chord between *C* and *G*. The panels *BC* and *ab* are longer, while *GH* and *hk* are shorter than the others. The inclination

of the end posts *Ba* and *Hk* is the same as the diagonals *Cd*, *De*, etc. *BC* and *ab* are usually made of the same length, as also *GH* and *hk*; then the inclination of *Bb* is the same as that of *Hh*. The truss opposite the one shown in elevation is the same in form and dimensions, but its ends are reversed as seen in the plan.

The stresses for such a truss are computed by the same methods as if the truss were symmetrical, although of course the inequality of the panels and loads makes the numerical work more laborious. The dead load at *b* should be one-half of that on *ab* plus one-half of that on *bc*, and similarly for all other panel points. The stresses are to be computed for all the members throughout the truss, since members in the right hand part do not correspond to those in the left.

Prob. 95. Let the dimensions for Fig. 79 be as follows: span = 146 feet, depth = 24 feet, $BC = ab =$ 22 feet $10\frac{1}{2}$ inches, $GH = hk =$ 13 feet $7\frac{1}{2}$ inches, all other panels = 18 feet 3 inches. Compute the maximum and minimum stresses due to a dead load of 600 pounds (one-fourth on the upper chord), and a live load of 1 700 pounds per linear foot per truss.

ART. 68. THE LATERAL BRACING.

The method of finding the wind stresses in the lateral bracing has been given in Arts. 42 and 47. This is in general made stronger than the wind stresses require, as it serves to stiffen the bridge laterally under the shock of moving trains. The wind stresses, however, are easily computed and may serve as a guide in proportioning the sizes of the members.

The upper lateral bracing of a truss is almost universally made with diagonal ties and normal struts. In the lower bracing the floor beams act as struts and diagonal ties are inserted which need to be much larger than for the upper bracing, since the wind pressure on the train is carried by the track directly to them.

As previously pointed out the stresses in the chords due to wind are not generally considered, but it is not logical that this should be the case. Art. 43 indicates that the limits of stress will be considerably increased by considering the wind, and this increase will be usually greater in railroad bridges. Further the initial tension produced by screwing up the lateral bracing brings stresses on the chords similar in character to some of those produced by wind. It is, therefore, to be recommended that specifications should require that maximum and minimum stresses due to dead and live loads should be modified by the wind stresses, so as to obtain the true greatest and least stresses possible under all combinations of conditions.

Prob. 97. Find the wind stresses for the lateral system in Fig. 79 due to a horizontal wind pressure of 35 pounds per square foot on trusses and train, estimating the area of the trusses by the approximate rule in Art. 42 and taking the train surface as 10 square feet for each linear foot of bridge.

ART. 69. THE ECONOMIC DEPTH OF TRUSSES.

The question of the proper depth of a truss is an interesting one. A through truss greater in span than 80 feet should have upper lateral bracing, and this requires a clear head room above the rail of at least 18 feet. Frequently the depths of deck bridges are determined by local considerations, such as the clear water way for the passage of boats below, the depth of adjacent spans, the expense of approaches, and even by æsthetic reasons.

The economic depth of a truss is that depth which renders the quantity of material, under the given specifications, a minimum. This depth is different for each kind of truss and for each class of specifications, and when these are the same it depends upon the panel length, the style of floor, and many other things besides

the span. The economic depth may be found, and in general has been found by bridge builders, by making several designs of trusses for different depths, the span and other conditions remaining the same. As in all cases of an algebraic minimum, slight variations from the true economic depth do not sensibly alter the quantity of material, so that a few trials will serve to determine its value.

The economic depth in general may be said to vary between one-fifth and one-eighth of the span, although for very long spans, as 500 feet, this gives too great a depth. Long panels are in general more advantageous than short ones in lessening the quantity of material.

A great many theoretic investigations as to economic depth have also been made. Most of these are of little value on account of assumptions which limit the investigation in some particulars, or exclude points which an actual design must include. The most complete and valuable investigation is that of DuBois, in the 'Transactions of the American Society of Civil Engineers' for May, 1887, to which the student is referred, and his particular attention directed to the numerous causes which effect the economic depth.

As an example of an approximate investigation let it be required to find the economic depth for the king-post truss in Fig. 80, the span being l, the depth d, the load P, and T and C being the tensile and compressive unit-stresses to be used for proportioning the members.

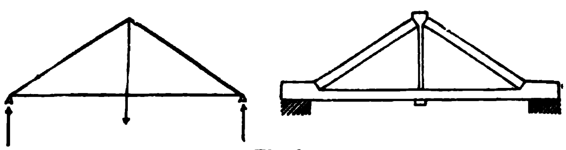

Fig. 80.

The stress in the vertical tie is P, its area $\frac{P}{T}$, and its volume $\frac{Pd}{T}$. The stress for the lower chord is $\frac{Pl}{4d}$, its area $\frac{Pl}{4Td}$, and its